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AN EXPERIMENTAL STUDY

OF THE

Resistances to the Flow of Water in Pipes.

THESIS

PRESENTED TO THE UNIVERSITY FACULTY OF CORNELL
UNIVERSITY FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

BY

AUGUSTUS VALENTINE SAPH

AND

ERNEST WILLIAM SCHODER.

JUNE, 1903.

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JUNE, 1903.

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AMERICAN SOCIETY OF CIVIL ENGINEERS.

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PAPERS AND DISCUSSIONS.

This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

AN EXPERIMENTAL STUDY OF THE RESISTANCES TO THE FLOW OF WATER IN PIPES.

By AUGUSTUS V. SAPH, ASSOC. AM. SOC. C. E., and ERNEST W. SCHODER, JUN. AM. SOC. C. E.

TO BE PRESENTED SEPTEMBER 2D, 1903.

Before proceeding to the presentation and discussion of the experimental data upon which this paper is based, it may be of interest to describe briefly the reasons which have led the writers to make the treatment as broad as the title indicates.

As graduate students in Cornell University, the writers had been associated in experimental hydraulics for a year and a half before the investigations to be described were begun. During that time they had experienced a number of surprises in an investigation on the effect of curvature on the flow of water in pipes,* and the necessity of refraining from jumping at conclusions, with little or no experimental evidence, had impressed them more and more as the work advanced.

In 1900, Gardner S. Williams, M. Am. Soc. C. E., Engineer in Charge of the Hydraulic Laboratory of Cornell University, made

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NOTE.—These papers are issued before the date set for presentation and discussion. Correspondence is invited from those who cannot be present at the meeting, and may be sent by mail to the Secretary. Discussion, either oral or written, will be published in a subsequent number of *Proceedings*, and, when finally closed, the papers, with discussion in full, will be published in *Transactions*.

* *Transactions*, Am. Soc. C. E., Vol. XLVII, p. 295.

an investigation,* from which one of the most interesting conclusions drawn was that the power of the velocity, with which the loss of head in water pipes varies, is dependent on the diameter of the pipe, being about 1 for very small tubes and increasing to about 2 for large pipes such as used ordinarily in the practice of the civil engineer.

For pipes below ⅛ in. in diameter, the value seemed to drop rapidly, while for pipes above that diameter the plotting showed a very gradual rise. However, the experimental results in the region where the most rapid change in the power seemed to occur were too few to give that feeling of entire confidence which is evident in the case of most of the commonly accepted laws of Nature. The desirability of further evidence was first pointed out to the writers by Professor Williams, and it is due to his interest in the matter, and to the true university spirit displayed by the College of Civil Engineering of Cornell University, that the writers were enabled to undertake experiments which, whatever their final aim became, had, as their primary object, the determination of the law governing the flow of water in small pipes.

The writers entered upon the investigation with the feeling that the small sizes of the pipes to be experimented upon would in no respect vitiate the results, from an engineering point of view. As will be shown later, the nature of the apparatus and the method of experimentation were to be such as to secure a high degree of accuracy in all the measurements. The writers, in their studies on the effect of curvature, had reached the conclusion that, in the manner of flow, there is practically no difference between a 2-in. and a 30-in. pipe. They expected, therefore, that much might be learned from the results of a comprehensive set of experiments on small pipes.

In addition to whatever light might be thrown upon the general subject of the flow of water in pipes, as ordinarily understood, the writers saw here an excellent opportunity to pursue their studies on the effect of the temperature of the water upon the loss of head, and hence upon the discharging capacity of pipes. The accepted idea in this matter is that the effect is negligible in pipes of commercial size. The writers had been led to question this, from their work on a 2-in. pipe,† and they saw in the experiments to be described a means of forming an idea concerning, or even of predicting the effect in large pipes.

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* *Journal* of the Association of Engineering Societies, Vol. XXVI (Mar., 1901), p. 185.

† *Transactions*, Am. Soc. C. E., Vol. XLVII, p. 313.

Soon after beginning the experiments it was found that the power of the velocity, with which the loss of head varies, remains almost constant for the fifteen pipes of different sizes, but all of the same material (seamless-drawn brass); or, in the formula,

$$H = m V^n,$$

where H is the loss of head and V the mean velocity of the water in the pipe, the value of n was found to remain nearly constant for all the pipes. This somewhat astonishing fact appeared at first to be irreconcilable with the results of the investigation by Professor Williams; but, fortunately, the writers' experiments covered a considerable range of velocities, and it was soon found that the group of experiments on small brass and glass pipes (by Jacobson), upon the indications of which Professor Williams, with considerable justification, had placed great confidence, were all made with velocities below the "critical velocity," in which region the value of n is about 1. Hence the results were not comparable with results of experiments made under ordinary, or "above critical velocity," flow. This phenomenon will be discussed more fully later.

The fact that the value of the exponent of V was found to be almost constant for pipes ranging from $\frac{1}{10}$ in. to 2 ins. in diameter would have been sufficient of itself to warrant an investigation into the indications of reliable experiments on large, smooth pipes; for the smallest of these sizes bears the same relation to the largest as a 2-in. pipe bears to a 40-in. main (see Fig. 1, Plate XXXVII). If this were the only argument for entering on such a study, the writers feel that they would well deserve the flood of criticism which is ever threatening those venturous persons who presume to affirm that the same laws of Nature control the flow of water in the smallest pipes in the laboratory and in the largest supply mains running over hill and dale. In this paper it is aimed to present a few additional arguments which may serve to make such an affirmation appear a little less ridiculous than heretofore.

After the experiments had progressed far enough, and several different temperatures of water had been experimented upon for each of the pipes, it became evident that there is nothing to support the notion that the effect of temperature is negligible in large, smooth pipes. In fact, the indications were found to point to such a considerable effect on pipes of large size that the limits of error which most hydraulic engineers would assign to the best formulas in use (all of which neglect temperature effect) are exceeded several times over, owing to the temperature factor alone.

The results of the brass-pipe experiments showed such harmonious relations that the writers were led to undertake some experiments on pipes which were less ideal and more practical. For this purpose five galvanized-iron pipes, ranging from ¼ in. to 1 in., commercial sizes, were chosen. While these experiments were conducted with the same care and precision as those on the brass pipes, and while each pipe by itself gave a consistent set of results, yet, between these results for the different pipes, there was a surprising lack of harmony, if it be assumed that all pipes were of the same degree of roughness. In short, the value of n, the exponent of V, in the formula $H = m\ V^n$, was found to vary so much for pipes nominally and apparently of the same kind that the writers saw here a consideration which might throw considerable light on the vagaries of the much-discussed, recorded pipe-experiments of modern and ancient times.

It may be well at this point to give a few reasons why the formula $H = m\ V^n$ is used. In the first place, the very fact that in the Chezy formula, $V = C \sqrt{r\ s}$, or, in the familiar textbook formula,

$$H = 4 f \frac{l}{d} \frac{V^2}{2\,g},$$

the values of C and f are different for different velocities, shows that the power is something other than 2, if there is, indeed, any exponential law which is applicable.* And the writers venture to say that, up to the present time, there has not been evolved any theory which will assist in answering this question.

To be sure, there seems to be a more or less firmly grounded notion among some hydraulicians that the laws of gravity decree that the hydraulic slope or the loss of head must vary as the square of the velocity. The fallacy of this has been shown very clearly by Professor I. P. Church,† Assoc. Am. Soc. C. E., who emphasizes the fact that the question concerns primarily the relation between the velocity and the resistances, or "friction," and through this, as an entirely dependent matter, the relation between the velocity and the hydraulic slope. An illustration may make the point clearer. It is quite possible, by the use of compressed air, to maintain a high velocity in a very long pipe laid on an absolutely level grade with just enough head of water at the entry to keep the pipe full. Now, it is at once apparent that the law of a falling body has nothing to do with the relation existing between the pressure in the compressed-air chamber and the velocity of the water in the pipe. In fact, the force of gravity alone, in this case, would cause no flow at all.

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* See discussion by Allen Hazen, M. Am. Soc. C. E., in *Journal* of the Association of Engineering Societies, Vol. XXVI (1901), p. 163.

† In *Engineering News*, Vol. XLVI. p. 332.

In the absence of any theory, the only thing to do is to study the indications of experiments of known accuracy. These should embrace a wide range of velocities and several different degrees of roughness. The records of many such experiments have been before the engineer for a number of years, as have been also the records of investigations dealing with the question of exponents other than 2 for the velocity, or other than $\frac{1}{2}$ for the slope and the diameter (or the hydraulic radius). Unfortunately, the formulas with fractional exponents, which have been proposed from time to time, have been regarded generally as approximations, little, if any, closer to the truth than the standard Chezy formula, and far less easy to apply. In fact, some engineers have been led to question whether or not water flows in a pipe according to any definite determinable laws whatsoever.

Now, the writers have investigated, by the method of logarithmic plotting of the observed losses of head with the corresponding velocities, experiments in which the velocity ranges from a fraction of a foot per second up to 44 ft. per second. Their own experiments, almost 800 in number, and including twenty-three kinds of pipe and hose, cover a range of velocity from $\frac{1}{10}$ ft. per second to 22 ft. per second. In all these experiments, and especially in those of known accuracy in all measurements, the logarithmic plottings yield straight lines. The slopes of these lines differ considerably among themselves, or, in other words, the losses of head vary as various powers of the velocities,* but, whatever the slope of the line, each pipe appears to have a definite law, expressible in the form,

$$H = m\,V^n.$$

The values of m and n are constant for a given pipe in a given condition for all velocities. The "coefficient of roughness" is entirely provided for by them. These appear to be scientific facts, and the writers felt that they had no choice as regards the formula to be used. As has been stated already, there is no existing theory which is applicable; but the experimental evidence is quite conclusive.

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* *Transactions*, Am. Soc. C. E., Vol. XLIX. p. 145.

It must not be inferred, from what has been stated, that the writers are opposed to the Chezy and like formulas, or, on the other hand, that they uphold the forms with fractional exponents for practical uses; but, for purposes of investigation, it would be most irrational to start with a formula which is known to be only an approximation, especially when the exact formula is just as easy to treat. But, after such an investigation is completed, on the actual showings of the experiments, it will be easy and quite proper to look for approximate forms which may be more convenient for the engineer to use. On this point, however, more can be said, and to better purpose, after the presentation of the experimental evidence.

Enough has been stated to indicate that the scope of the investigation, as planned originally, has been enlarged considerably, and for such reasons that the writers feel that the whole subject of the flow of water in pipes may be considered open for a treatment in which the aim is to present the facts, in their relations to one another, in a thoroughly rational and comprehensive manner. It is the purpose, as far as possible, to make all plottings bird's-eye views of the whole subject, so that the experimental results may show at a glance the probable laws which too often have been obscured by misleading approximate formulas, and unfortunate attempts to make the results of the roughest kind of measurements fit these formulas. In the general plotting, many results of such rough measurements have been included. In fact, the idea has been to consider all experiments which are on record in any recognized source of information. But it is believed that the method of plotting has reduced to the greatest possible extent the question of individual judgment, and has allowed each experimental result to have its merited influence in controlling the final conclusions.

DESCRIPTION OF APPARATUS.

The general plan of the pipe system experimented upon is shown in Fig. 1. The pipes were located in the basement of Lincoln Hall, the principal building of the College of Civil Engineering, Cornell

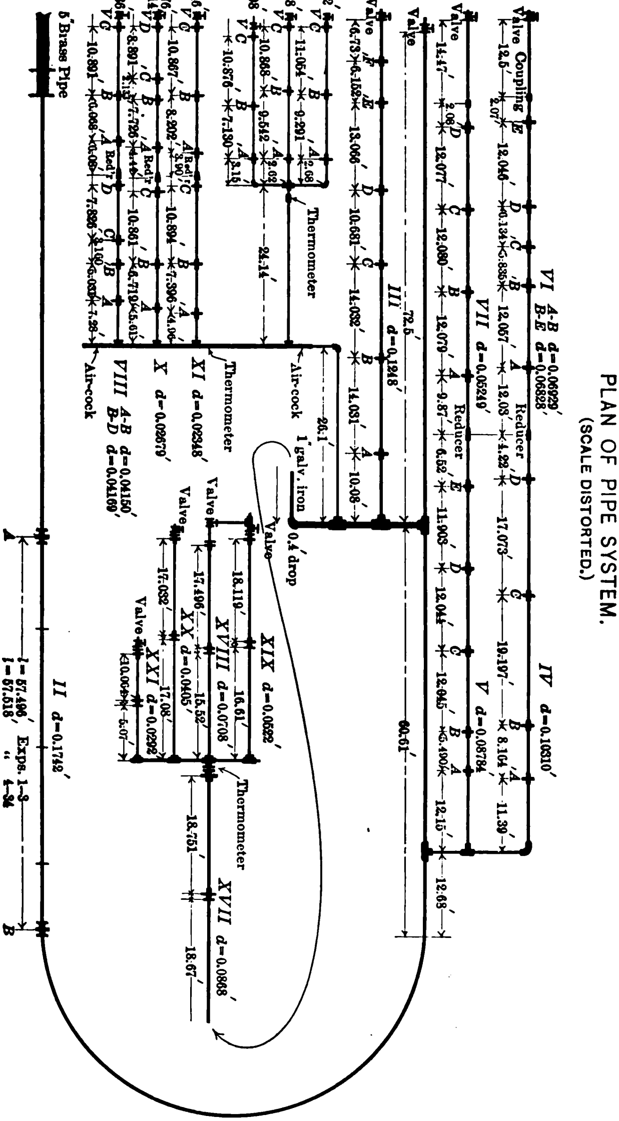
PLAN OF PIPE SYSTEM.
(SCALE DISTORTED.)
IV d=0.10310′
V d=0.08784′
VI A-B d=0.06929′
B-E d=0.06828′
VII d=0.05249′
III d=0.1248′
XVI d=0.00892′
XV d=0.01258′
XIV d=0.01496′
XIII d=0.01846′
XII A-B d=0.02176′
B-D d=0.02214′
IX d=0.03136′
XI d=0.02348′
X d=0.02679′
VIII A-B d=0.04150′
B-D d=0.04169′
XIX d=0.0632′
XVIII d=0.0708′
XX d=0.0405′
XXI d=0.0292′
XVII d=0.0868′
II d=0.1742′
Valve
Coupling
Reducer
Red'r
Thermometer
Air-cock
1″ galv. iron
0.4′ drop
5″ Brass Pipe
72.5′
80.61′
26.1′
24.14′
l=57.496′ Exps. 1-3
l=57.518′ " 4-34
A
B
C
D
E
F

FIG. 1.

University. The system was fed through the 5 and 2-in. pipes. The former extended from an equalizing cylinder, which could be supplied either from a tank in the attic (the supply to which was controlled by a float-valve) or directly from the campus mains.* The pressures in the cylinder, under static conditions, were approximately 20 and 30 lbs. from the attic tank and the main, respectively.

All elbows, tees and auxiliary pipes are of galvanized iron except the plain iron steam-pipe header from which extend Pipes *VIII* to *XIII* and a pipe feeding Pipes *XIV*, *XV* and *XVI*. All piezometric fittings are of brass. Experimental Pipes *II* to *XVI* are of seamless-drawn brass, and Pipes *XVII* to *XXI* are of ordinary commercial, galvanized-iron stock. The fittings on all the pipes except Pipe *II*† are described herein.

Piezometric Fittings.—The individual pipe lengths of Pipes *III* to *XVI* are connected by piezometric couplings of special design. Each coupling is in two pieces which screw together in such a way that when the pipe is ready for experimentation the pressure is transmitted to an annular chamber through a continuous slit or break in the pipe $\frac{1}{100}$ in. wide. The details of the construction are shown in Fig. 2, and in Fig. 1 of Plate XXXVII. The latter is from a photograph of piezometric couplings on Pipe *III* and Pipe *XVI*, and shows also the comparative sizes of these two pipes. The lathe work inside these couplings is practically perfect, while the differences between the inside diameters of the pipe and the coupling where the faced end of the pipe screws up against a shoulder are generally not more

SECTION OF PIEZOMETER NO. *III F*

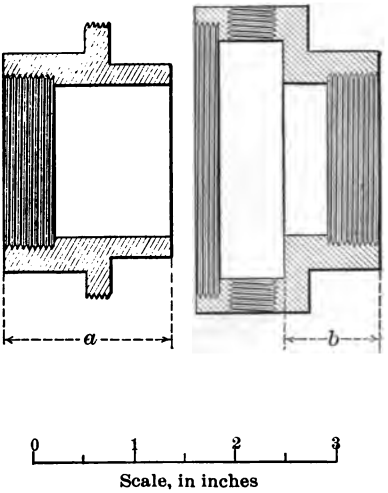

FIG. 2.

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* The cylinder and the controlling valves are shown in Fig. 119, *Transactions*, Am. Soc. C. E., Vol. XLVII, p. 297. The cycloidal mouthpieces between the cylinder and the 5-in. pipe and between the latter and the 2-in. pipe are described on p. 20 of the above-mentioned volume, and also on p. 180 of the *Journal* of the Association of Engineering Societies, Vol. XXVI, 1901, while Plate 25 in the latter gives a longitudinal section of this part of the apparatus.

† The piezometer fittings on Pipe *II* have been illustrated on p. 17 and in Plate IX, Fig. 1, *Transactions*, Am. Soc. C. E., Vol. XLVII.

PLATE XXXVII.
PAPERS, AM. SOC. C. E.
MAY, 1903.
SAPH AND SCHODER
ON FLOW OF WATER IN PIPES.

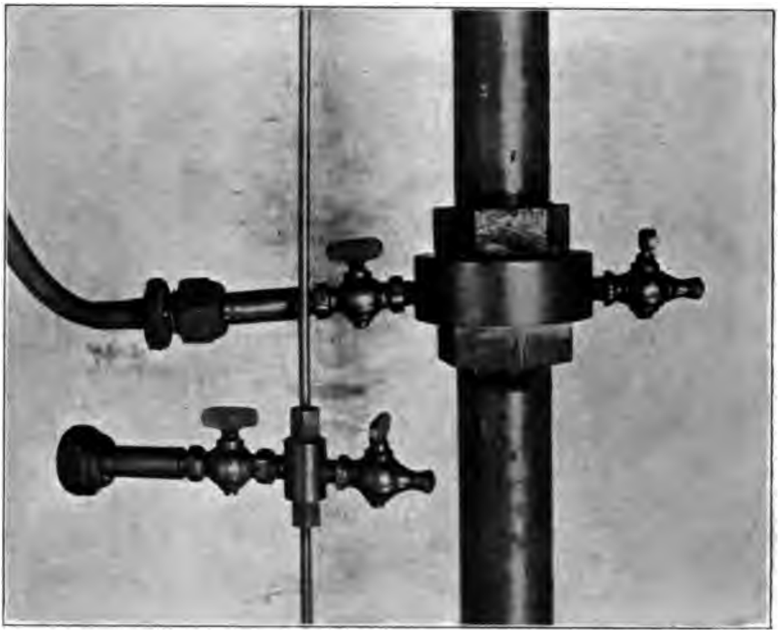

Fig. 1.—Piezometer Couplings. Pipes *III* and *XVI*.

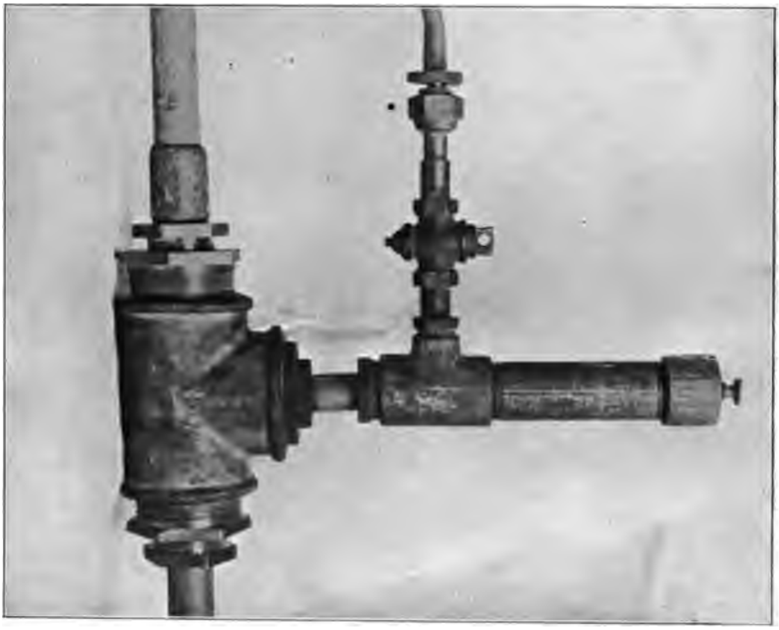

Fig. 2.—Piezometer Chamber on Pipe *XVIII*.

than $\frac{1}{1000}$ in., or the smallest amount which can be measured accurately with the ordinary micrometer calipers (see Table No. 1). Before the couplings were turned, the ends of all the pipes had been measured and the pipes matched. All threads were lathe-cut. In general, the pipes, when set up, left nothing to be desired, as far as nearly ideal conditions are concerned. The writers feel that without the excellent machine-work done by Mr. C. D. Cass, Mechanician to the Hydraulic Laboratory, the value of the experimental accuracy would have been very small. They desire to acknowledge their indebtedness to Mr. Cass for his accurate and conscientious work.

On account of the roughness of the galvanized pipes, as well as on account of the variable diameter, it was deemed best to avoid any form of joint at the point where the pressure was to be taken off. Accordingly, four ⅛-in. holes, 90° apart, were drilled through the pipe walls 3½ ins. from the ends of each experimental length. For the two smallest pipes the holes were $\frac{3}{32}$ in. in diameter, and the distance from the ends was 2½ ins. The drill passed through both walls of the pipe at one setting. Thus, only two of the four holes had any burr inside. This was removed carefully with a rounded file. These holes in the finished piezometer fitting communicated to a chamber formed by a large tee slipped over the pipe and provided with a pair of brass bushings at each end. Between these bushings, and on the pipe, some candle wicking soaked with tallow was wound. When the smaller bushings were screwed up, the fitting was water-tight. A photograph of one of these fittings on Pipe *XVIII* (¾-in. galvanized iron) is shown in Fig. 2, Plate XXXVII.[*] In order to avoid any error due to unequal diameters at the piezometers, all the experimental lengths were turned end for end, after the first set of experiments had been completed, and a second series was run with the reversed arrangement. In this way, also, the effect of any slight differences in the conditions of the flow at the two piezometers was eliminated. The up-stream, or entry, length of pipe was connected to the experimental length by an ordinary coupling. The ends of the pipe, however, were faced, and the threads were cut so that a butt joint was secured, avoiding the disturbing effect of the usual coupling on wrought-iron pipe where the ends do not come together.

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[*] The air-chamber has a section like that shown on p. 17, *Transactions*, Am. Soc. C. E., Vol. XLVII.

TABLE No. 1.—Diameters (in inches) of Brass Pipes and Piezometers.

No. of Pipe.	Length No. 1.	Piezometer A.	Length No. 2.	Piezometer B.	Length No. 3.	Piezometer C.	Length No. 4.	Piezometer D.	Length No. 5.	Piezometer E.	Length No. 6.	Piezometer F.	Length No. 7.
III............	1.4976 1.4986	1.498	1.4984 1.4976	1.498	1.4986 1.4983	1.498	1.4979 1.4962	1.498	1.4978 1.4984	1.498	1.4960 1.4988	1.498	1.4988 1.4980
IV............	1.2465 1.2463	1.246	1.2376 1.2378	1,237	1.2361 1.2368	1.237	1.2378 1.2362	1.237	1.2378 1.2384	....	....	....	....
V............	1.0693 1.0686	1.069	1.0531 1.0541	1.054	1.0543 1.0543	1.054	1.0539 1.0557	1.054	1.0539 1.0553	1.054	1.0531 1.0530	....	....
VI............	0.8321 0.8310	0.8315	0.8313 0.8318	0.8315	0.8197 0.8201	0.819	0.8201* 0.8201	0.819	0.8191 0.8189	0.819	0.8189 0.8344	....	....
VII............	0.6307* 0.6303	0.630	0.6297 0.6293	0.630	0.6293 0.6299	0.630	0.6308 0.6305	0.630	0.6307 0.6301	....	....	....	....
VIII............	0.4976* 0.4982	0.498	0.4982 0.4977	0.498	0.5008 0.5006	0.500	0.5008 0.4998	0.500	0.5006 0.5008	....	....	....	....
IX............	0.3764 0.3766	0.376	0.3763 0.3768	0.376	0.3750 0.3766	0.376	0.3766 0.3768	....	....	....	....	....	....
X............	0.3207 0.3213	0.3215	0.3218 0.3221	0.3215	0.3215 0.3217	0.3215	0.3217 0.3213	....	....	....	....	....	....
XI............	0.2817 0.2817	0.282	0.2813 0.2823	0.282	0.2821 0.2811	0.282	0.2811 0.2813	....	....	....	....	....	....
XII............	0.2608 0.2613	0.261	0.2622* 0.2602	0.261	0.2652 0.2644	0.266	0.2652* 0.2654	0.266	0.2644 0.2661	....	....	....	....
XIII............	0.2217 0.2209	0.222	0.2213 0.2220	0.222	0.2217* 0.2220	0.222	0.2217 0.2211	....	....	....	....	....	....
XIV............	0.1807 0.1789	0.180	0.1801 0.1801	0.180	0.1800 0.1799	0.180	0.1799 0.1787	....	....	....	....	....	....
XV............	0.1507 0.1492	0.151	0.1511 0.1522	0.151	0.1506 0.1526	0.151	0.1526 0.1514	....	....	....	....	....	....
XVI............	0.1057 0.1081	0.107	0.1071 0.1083	0.107	0.1055 0.1073	0.107	0.1073 0.1081	....	....	....	....	....	....

Note.—The first of the two diameters under each length is for the up-stream end in the usual arrangement. This is also the marked end, except where an asterisk denotes that the unmarked end is up stream.

Measurements of Diameters.—From the ends of all the brass and galvanized-iron pipes there were cut sample rings from ⅛ to ¼ in. long. Before cutting off the rings in the lathe, the ends of the pipe were faced, the tool being started on the inside wall of the pipe, thus securing a sharp, clearly defined edge, and avoiding entirely the burr which would result if the tool were started on the outside.

Four diameters were measured on each ring. The rings from the brass pipes were placed on the bed of a dividing engine, and readings were taken as the inner edges appeared under the cross-hair of the microscope. The bed advanced 1 mm. for each turn of the screw, and one two-hundredth part of a turn could be read directly from a graduated disc on the screw. The rings from the galvanized pipes were measured with small micrometer beam-calipers reading to one-thousandth of an inch.

The means of the four measured diameters on each ring, corresponding to the mean diameters of the ends of the brass pipes, are given in Table No. 1. The numbers refer to the position of the individual pipe lengths in the experimental arrangement, the lengths being numbered consecutively, beginning at the up-stream end. In a number of cases a single piece of pipe was cut into two or three lengths. Hence the question as to the uniformity of diameter can be answered by direct evidence. The following list shows what lengths were cut from the single factory lengths.

III.—No. 7 was cut from No. 6
IV.— " 5 " " " " 2
V.— " 2 " " " " 5
VI.— " 4 " " " " 3
VI.— " 6 " " " a piece, the remainder of which was not used.
VII.—No. 5 was cut from No. 1
VIII.— " 2 " " " " 1
VIII.— " 5 " " " " 4
VIII.— " 3 " " " " 4
IX.— " 1 " " " " 2
IX.— " 4 " " " " 3
X.— " 1 " " " " 2
X.— " 4 " " " " 3
XI.— " 1 " " " " 2

XI.—No. 5 was cut from No. 4
XI.— " 3 " " " " 4
XII. — " 1 " " " " 2
XIII.— " 1 " " " " 2
XIII.— " 4 " " " " 3
XIV.— " 1 " " " " 2
XIV.— " 4 " " " " 3
XV.— " 1 " " " " 2
XV.— " 4 " " " " 3
XVI.— " 1 " " " " 2
XVI.— " 4 " " " " 3

If a pipe was cut into three pieces, the shortest piece was cut off first. All pieces were cut from the unmarked end of the factory length. The unmarked end of the original pipe became the unmarked end of the first piece cut off. Hence, the marked end of a short pipe cut from a factory length represents a section in the middle of the latter.

TABLE No. 2.—DIMENSIONS OF GALVANIZED PIPES.

No. of pipe.	Length, in feet.	DIAMETERS, IN INCHES.					
		Nominal.	Standard.	BY CALIPERS.		Water-filling method.	Accepted diameter.
				Marked end.	Unmarked end.		
XVII............	18.751	1-in.	1.048	1.046 1.050*	1.037 1.044	1.031 1.037	1.036 1.042†
XVIII...........	17.496	¾-in.	0.824	0.789 0.850	0.799 0.850	0.815 0.849	0.805 0.850†
XIX.............	18.119	½-in.	0.623	0.627 0.631*	0.628 0.631	0.627 0.622	0.627 0.626†
XX..............	17.032	⅜-in.	0.494	0.493 0.489	0.502 0.482	....	0.498 0.486†
XXI.............	10.064	¼-in.	0.364	0.363 0.355*	0.345 0.345	....	0.354 0.350†

NOTE.—The lengths given in the table are for the experimental sections. For other lengths see Fig. 1.

* The marked ends are also the up-stream ends in the first or unreversed arrangement, except where an asterisk denotes the opposite.

† The experimental section.

In Table No. 2 are given the mean diameters of the galvanized pipes. In addition to the end caliperings, Pipes *XVII*, *XVIII* and *XIX* were measured by the water-filling method, special precautions being taken to avoid the adhesion of air to the walls of the pipe. The pipe was held vertically, and a stiff wire with a piece of cloth tied to the lower end was inserted until the cloth was near the bottom of the pipe. The pipe was then filled, and the wire was drawn up with a rotary motion. The pipe was finally filled flush, and the contained water was caught and weighed to the nearest $\frac{1}{100}$ lb. Each pipe length was filled three times. Allowance was made for the small quantity of water which remained adhering to the inside walls of the pipe, a determination having been made.

Measurements of Lengths.—All lengths were measured with an accurate steel tape, and were checked by independent measurements. They are given in Fig. 1 and in Table No. 3. For the experimental sections, they are, in all cases, between the piezometric slits for the brass pipes, and between the piezometric holes for the galvanized pipes.

Methods of Experimentation.—All pipes were laid straight, and, where necessary, intermediate supports were used between the hangers on which the whole pipe system was supported. On account of their very small sizes, Pipes *XIV*, *XV* and *XVI* were laid on a board, and held straight by tacks. The hangers were levelled up carefully.

As shown in Fig. 1, there are fourteen valves to control the flow in the various experimental lines, several of which have two different sizes of pipes. It will be noticed that there is in all cases a considerable length of pipe up stream from the first piezometer, or Piezometer *A*. In fact, the writers had agreed that it is desirable to have an entry length of approximately two hundred diameters, in order to allow the flow to become normal. While this allowance is ample, it does not appear to be excessive. A length of one hundred diameters is, in the writers' opinion, hardly enough to allow the disturbances due to a curve at entry to die down fully.*

In order to get as large a range of velocities as possible, the downstream sections of Pipes *VIII* to *XVI* were removed and replaced again from time to time. On several occasions Pipes *VI*, *VII*, *IX*, *XII* and *XIII* were entirely removed, in order to increase the discharge of the larger pipes ordinarily feeding into them.

***Transactions*, Am. Soc. C. E., Vol. XLVII, p. 317.

TABLE No. 3.—BRASS PIPE—LENGTHS OF SECTIONS AND DIAMETERS.—SEE FIG. 1.

Pipe.	Section.	Length, in feet.	Accepted diameter, in feet.	Pipe.	Section.	Length, in feet.	Accepted diameter, in feet.
II.	*A-B*	57.496	0.1742	*IX.*—1	Reducer-*A*	6.08	0.03137
		57.518		2	*A-B*	6.068	
III.—1	Entry-*A*	10.08	0.12484	3	*B-C*	10.891	
2	*A-B*	14.031		4	*C*-Valve.	1.02	
3	*B-C*	14.032		*X.*—1	Entry-*A*	5.61	0.02679
4	*C-D*	10.681		2	*A-B*	6.719	
5	*D-E*	13.066		3	*B-C*	10.861	
6	*E-F*	6.152		4	*C*-Reducer.	1.075	
7	*F*-Valve.	6.73		*XI.*—1	Entry-*A*	4.96	0.02347
IV.—1	Entry-*A*	11.39	0.10387	2	*A-B*	7.396	
2	*A-B*	8.164	0.10311	3	*B-C*	10.894	
3	*B-C*	19.197		4	*C*-Reducer.	1.06	
4	*C-D*	17.073		*XII.*—1	Reducer-*A*	4.44	0.02176
5	*D*-Valve.	4.22		2	*A-B*	7.723	
V.—1	Entry-*A*	12.15	0.08908	3	*B-C*	2.133	0.02209
2	*A-B*	5.490	0.08785	4	*C-D*	8.891	
3	*B-C*	12.045		5	*D*-Valve.	1.04	
4	*C-D*	12.044		*XIII.*—1	Reducer-*A*	3.90	0.01846
5	*D-E*	11.903		2	*A-B*	8.202	
6	*E*-Reducer.	6.52		3	*B-C*	10.867	
VI.—1	Reducer-*A*	12.03	0.06930	4	*C*-Valve.	1.02	
2	*A-B*	12.057		*XIV.*—1	Entry-*A*	3.15	0.01498
3	*B-C*	5.835	0.06831	2	*A-B*	7.130	
4	*C-D*	6.134		3	*B-C*	10.876	
5	*D-E*	12.046		4	*C*-Valve.	1.02	
6	*E*-Coupling.	2.07	0.06889	*XV.*—1	Entry-*A*	2.62	0.01261
VII.—1	Reducer-*A*	9.87	0.05251	2	*A-B*	9.542	
2	*A-B*	12.079		3	*B-C*	10.868	
3	*B-C*	12.080		4	*C*-Valve.	1.02	
4	*C-D*	12.077		*XVI.*—1	Entry-*A*	2.68	0.00893
5	*D*-Enlargement	2.08		2	*A-B*	9.291	
	to valve.	14.47	....	3	*B-C*	11.054	
VIII.—1	Entry-*A*	7.28	0.04149	4	*C*-Valve.	1.02	
2	*A-B*	5.031		....	....	....	....
3	*B-C*	3.160	0.04172	....	....	....	....
4	*C-D*	7.826		....	....	....	....
5	*D*-Reducer.	1.060		....	....	....	....

The differences of pressure between the piezometers were measured by either mercury or water gauges, of the differential type, the pressures being transmitted from the piezometers to the gauge columns by heavy, three-ply, cotton-insertion, rubber tubing. The scales of these gauges are divided into double centimeters and tenths, the hundredths being estimated. All gauges were supplied with riders to facilitate accurate reading of the menisci. The mercury gauges had three columns, and the water gauges had either two or three columns. Any of the gauges could be used as two-column gauges, if desired.

The water was discharged for measured intervals of time into a tank on one of three platform scales. One of these scales has a capacity of 2 500 lbs., and is accurate to the nearest pound. Another has a capacity of 350 lbs., and is accurate to $\frac{1}{10}$ lb. The third is sensitive to $\frac{1}{100}$ lb. for weights not exceeding 50 lbs. The two smaller scales were calibrated with standard weights in the possession of the College of Civil Engineering. The largest was tested by comparison.

Thus, it was possible to secure the desired accuracy for the measurement of the discharge of the smallest of the pipes as well as the largest. In general, the attempt was made to secure an average accuracy of 1 in 500 in all measurements, whether of weight, time or gauge differences. For the highest velocities, a much greater accuracy has been obtained, while for some of the very lowest velocities, the weight or gauge difference was so small as to render the result accurate within only about $\frac{1}{2}$ of 1%, or, very rarely, within only 1%; but, for the vast majority of the experiments, it is believed that the attempted accuracy has been secured.

For many of the first experiments, the time was taken from a record sheet of a chronograph connected to the diverting arrangements and to the standard clock of the Astronomical Laboratory. It was found that the error in using an ordinary well-regulated watch was in general only a small fraction of a second. In fact, by using a small reading glass, the $\frac{1}{5}$-second beats can be seen quite distinctly on the second hand of the ordinary watch. Hence, for all the later runs, the time was taken with an ordinary watch, and it is believed that the time element is well within the attempted limit of accuracy.

During the time in which the flow was being measured, the gauge was read from six to twenty times. When the pressure was taken from the attic tank the columns were quite steady, but when connected to the campus mains, there was considerable vibration, yet not enough to impair greatly the accuracy of the readings. The main pressure was used, generally, only for the highest velocities, and hence the greatest gauge differences. The gauges were tested under no-flow conditions before and after, and frequently during, every series of runs. In general, no trouble was experienced. The columns came quickly to a common level. This condition of affairs the writers attribute to the complete control of the apparatus, the excellence of hose and gauge connections, and the great care taken to free the whole pipe system from air before starting any experiment. The temperature of the water for the first experiments was taken in the tank after the runs.

For these cases the temperature of the water was within 1 or 2° Fahr., of the temperature of the room. For the later experiments on Pipes *VIII* to *XVI*, and for all the experiments on Pipes *XVII* to *XXI* (of galvanized iron), the temperature was taken with a thermometer inserted in a stuffing-box, and with its bulb well within the pipe. The positions of these thermometer stuffing-boxes are shown in Fig. 1. The temperature was taken at the beginning and at the end of each experiment, and the mean value used, the differences being small. For Pipes *II* to *VII* it was found that the temperature in the pipes was the same as that in the tank after filling, within less than half a degree. Hence, the latter was taken, except at low velocities, when a small jar full of the water was caught just before and after each run and the temperature was read immediately.

Methods of Reduction.—In reducing weights to velocities the weights per cubic foot of distilled water at the observed temperature were taken from a table by the late Hamilton Smith, Jr., M. Am. Soc. C. E. These values are known to be correct within $\frac{1}{10}$ of 1% for the water used in the experiments.

In reducing observed heads on the differential water gauges a correction is necessary on account of the weight of a column of air of a height equal to the observed difference and under an average pressure of, approximately, 1½ atmospheres. This amounts to a subtraction of approximately one five-hundredth of the observed differences. There is another correction on account of the difference between the temperature of the water in the gauge and that in the pipe, but this is very small, and is an uncertain amount, varying from nothing to a possible subtraction of one-six-hundredth of the observed difference. The correction actually applied, to cover these two items, is a subtraction of one five-hundredth of the observed difference for temperatures in the pipe below 60° Fahr.

For reducing observations on the mercury gauges, the "water equivalent" was obtained by direct comparison. A line of levels was run, and the elevation of the top of the attic tank above the zeros of the gauges was determined accurately. One observer measured the level of the surface of the water in the tank, while another took readings on the gauges in the basement. Eight determinations were made, with as many different heights of the water surface. The "water equivalent" was found to be 13.57 for mercury and water at 71° Fahr.

The eight determinations agreed within $\frac{1}{10}$ of 1 per cent. The weight of a column of air from the gauge to the attic was included in the calculation. Since the temperature of the room, and hence the specific gravity of the mercury in the gauges, was practically constant during the course of the experiments, the observed differences were corrected according to the specific gravity of the water flowing in the pipe. Thus all heads are reduced to water at the temperature of that in the pipe.

All reductions of weights to velocities, and of observed gauge differences to loss of head, in feet of water per 100 ft. of pipe length, have been checked by independent calculations. In addition, all results have been subjected to the test of two different methods of plotting, and any apparently wild cases have been recalculated. The writers have been pleased to find that the efforts to maintain a high degree of accuracy in all measurements have borne good fruit in the resulting indications.

Table No. 4 gives the data for the brass pipes, and Table No. 5 the data for the galvanized pipes.

BRASS PIPE V.

48	B-D	4.179	D-E	4.146	70.8°	3.087	† Heads read on mercury gauges.
49	"	3.721	"	3.699	70.1	2.887	
50	"	2.713	"	2.707	70.1	2.415	
51	"	2.275	"	2.288	70	2.179	
52	"	1.896	"	1.885	70	1.659	
53	"	1.137	"	1.119	70	1.438	
54	"	0.527	"	0.546	70	0.942	
55	"	0.185	"	0.182	70	0.517	
209	....	....	B-E	0.312	71.6	0.751	
210	....	....	"	0.250	71.6	0.672	
211	....	....	"	0.123	71.6	0.447	
212	....	....	"	0.085	71.6	0.359	
263†	B-D	25.20	D-E	25.22	70.8	8.609	
264†	"	17.94	"	17.80	70.5	7.099	
265†	"	13.56	"	13.44	70.6	6.039	
266†	"	9.28	"	9.14	70.6	4.864	Temp., Air 71°.
299†	"	25.86	"	25.68	61.4	8.535	
200†	"	18.26	"	18.20	61.6	7.044	
301†	"	11.08	"	11.01	61.6	5.285	
302†	"	3.90	"	3.81	61.6	2.896	
347	D-E	18.67	....	....	54.5	7.030	
348	"	13.72	....	....	54.5	5.926	
349	"	8.88	....	....	54.5	4.624	
350	"	4.01	....	....	54.5	2.927	
351	"	24.67	....	....	55.0	8.208	
725	C-D	19.87	....	....	41	7.054	
726	"	11.56	....	....	42.1	5.133	
727	"	28.49	....	....	37.2	8.503	
728	"	21.36	....	....	36.6	7.200	
729	D-E	20.17	....	....	40	7.016	
730	"	12.73	....	....	39	5.395	
731	"	5.49	....	....	39.1	3.845	
732	"	27.33	....	....	35.8	8.225	

TABLE No. 4—(*Continued*).

BRASS PIPE *IV*.

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
40			*B-D*	5.61	69.0°	4.097	† Heads were measured in mercury.
42	*B-C*	4.06	*C-D*	4.06	69.1	3.384	
43	“	2.88	“	2.88	69.0	2.794	
44	“	2.03	“	2.04	69.0	2.279	
45	“	1.39	“	1.40	69.0	1.813	
46	“	0.52	“	0.52	69.0	1.019	
47	“	0.17	“	0.19	69.1	0.549	
205			*B-D*	0.145	72.2	0.506	
206			“	0.109	72.1	0.429	
207			“	0.255	72.0	0.695	
208			“	0.371	71.7	0.867	
259†	*B-C*	15.03	*C-D*	14.92	70.1	7.365(?)	Evidently a mistake of 5 seconds in time. V=7.244 therefore used.
260†	“	10.91	“	10.77	73.2	5.975	
261†	“	8.12	“	7.92	72.8	5.052	
262†	“	10.96	“	10.82	71.0	5.994	
303†	“	15.41	“	15.54	62.0	7.135	
304†	“	11.11	“	11.34	62.2	5.959	
305†	“	7.73	“	7.92	62.2	4.839	
306†	“	4.63	“	4.73	62.1	3.637	
307†	“	1.89	“	1.93	62.1	2.154	
303	“	15.43			62.0	7.135	
304	“	11.19			62.2	5.959	
305	“	7.85			62.2	4.839	
306	“	4.67			62.1	3.637	
307	“	1.89			62.1	2.154	
342	“	11.44			54.9	5.946	
343	“	8.30			54.7	4.960	
344	“	5.45			54.6	3.890	
345	“	2.39			54.5	2.427	
346	“	14.39			54.7	6.771	
718	“	12.11			46.5	5.977	
719	“	8.00			43.2	4.698	
720	“	2.82			43.0	2.584	
721	“	14.53			37.2	6.516	
722	“	9.97			37.0	5.269	
723	“	4.99			38.0	3.542	
724		2.12			38.5	2.167	

Brass Pipe *VII.*

50........	*B–C*	31.09	....	....	70.1°	6.762	† Heads read in mercury.
51........	“	26.04	....	....	70	6.101	
52........	“	21.61	....	....	70	5.486	
53........	“	12.61	....	....	70	4.028	
54........	“	6.067	....	....	70	2.637	
55........	“	2.140	....	....	70	1.447	
56........	“	0.652	....	....	70.7	0.728	
195†........	*A–C*	77.88	*C–D*	78.10	69.3	11.372	
196†........	“	60.95	“	61.24	69.8	9.889	
197†........	“	46.97	“	47.11	69.7	8.560	
198†........	“	33.18	“	33.05	70.9	7.001	
199........	*A–D*	0.440	....	....	71.3	0.648	
200........	“	0.187	....	....	71.3	0.471	
201........	“	0.087	....	....	71.8	0.218	
380†........	*A–C*	78.29	*C–D*	77.04	72.0	11.375	After Experiment 201 Pipe *VII* was taken out and was allowed to remain empty. When set up again, the circulation through the piezometers was poor. Upon investigation, iron rust was found in the slits. These were easily cleaned..
381†........	“	79.35	“	78.45	47.5	10.940	
382†........	“	67.97	“	67.41	48.0	10.056	
383†........	“	52.77	“	52.37	56.1	8.836	
384†........	“	41.24	“	40.77	54.6	7.635	
385†........	“	31.52	“	30.95	53.2	6.525	
386†........	“	21.16	“	21.13	52.5	5.208	
387†........	“	10.70	“	10.77	53.0	3.521	
388........	“	0.342	“	0.348	85.8	0.538	
389........	“	0.834	“	0.831	84.0	0.877	Temp., Air 76° in 383 to 387.
390........	“	1.524	“	1.521	81.0	1.230	
391........	“	1.208	“	1.206	81.0	1.075	
392........	“	2.059	“	2.052	81.0	1.609	
393........	“	3.316	“	3.282	77.0	1.881	
469†........	“	79.38	“	78.10	40.8	10.686	Air, 71.6°.
470†........	“	53.83	“	52.48	47.5	8.708	
471†........	“	42.38	“	41.57	46.1	7.542	
472†........	“	32.77	“	32 37	45.4	6.512	
473†........	“	20.37	“	20.17	46.0	4.949	
474†........	“	10.43	“	10.36	46.6	3.361	Air, 64°.
475†........	“	81.76	“	80.35	38.2	10.745	
476†........	“	67.69	“	66.31	37.8	9.711	
477†........	“	49.12	“	48.45	37.5	8.098	
478†........	“	35.33	“	34.89	37.3	6.680	

TABLE No. 4—(*Continued*).

BRASS PIPE *VI*.

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
41	*C–D*	40.15	....	....	69.0°	9.316	† Heads read in mercury.
42	“	28.93	....	....	69.1	7.712	
43	“	20.47	....	....	69.0	6.368	
44	“	14.38	....	....	69.0	5.194	
45	“	9.84	....	....	69.0	4.133	
46	“	3.66	....	....	69.0	2.322	
47	“	1.28	....	....	69.1	1.252	
180†	*A–D*	56.22	*D–E*	53.18	69.3	11.477 For *D–E*.	11.146 Velocities in *A–B*.
181†	“	43.48	“	40.86	69.3	9.933	9.617
182†	“	30.93	“	29.23	70.3	8.212	7.975
183†	“	21.42	“	20.40	70.3	6.668	6.476
184†	*B–C*	10.54	“	9.94	70.6	4.445	4.317
185	“	59.97	....	....	69.3	11.477	
186	“	46.28	....	....	69.3	9.933	
187	“	32.84	....	....	70.3	8.212	
188	“	22.72	....	....	70.3	6.668	After Experiment 193, Pipe *VI* was taken down and remained empty for a time. Upon setting again, the circulation through the piezometers was poor. Iron rust was found in the slits. These were easily cleaned.
189	“	11.12	....	....	70.6	4.445	
190	*C–E*	1.56	....	....	70.9	1.501	
191	“	1.01	....	....	70.9	1.172	
192	“	0.722	....	....	71.1	0.962	
193	“	0.433	....	....	70.0	0.716	
399	*A–B*	30.11	....	....	51.5	7.705	
400	“	24.18	....	....	48.0	6.755	
401	“	16.71	....	....	47.0	5.470	
402	“	8.90	....	....	46.4	3.803	
403	“	1.58	....	....	46.6	1.404	
404	“	4.42	....	....	47.0	2.448	Air, 74°.
405	“	29.98	....	....	44.0	7.590	
406	“	13.19	....	....	44.0	4.720	
551†	*C–E*	58.40	....	....	36.3	11.055	
552†	“	45.72	....	....	36.2	9.638	
553†	“	34.72	....	....	37	8.223	
554†	“	24.99	....	....	37	6.841	
555†	“	11.54	....	....	37.6	4.352	
709†	*C–E*	38.60	....	....	79	9.382	
710†	“	39.72	....	....	53	9.114	
711†	“	56.74	....	....	48.7	11.125	
712†	“	58.90	....	....	38	11.131	
713†	“	27.25	....	....	45.1	7.270	

BRASS PIPE *IX.*

64........	*A–B*	73.36	....	....	69.1°	7.868 (?)	† Heads read in mercury. * Columns vibrating badly.
65........	"	59.15	....	....	69.5	6.777	
66........	"	39.29	....	....	69.6	5.371-	
67........	"	27.24	....	....	69.9	4.356	In Experiment 64 there was evidently a mistake of 5 lbs. in the weight. This was checked by Experiment 70. Velocity should be 7.652.
68........	"	15.06	....	....	70.1	3.119	
69........	"	6.38	....	....	70.5	1.928	
70........	"	73.05	....	....	71.0	7.631	
176†........	"	172.6	*B–C*	172.2	67.5	12.28	
177†........	"	117.5	"	117.7	67.5	9.884	
178†........	"	68.35	"	68.29	69.0	7.286	
179†........	"	40.76	"	41.04	69.3	5.334	
275†........	"	173.1	"	173.7	66.1	12.31	
276†........	"	115.6	"	115.8	65.0	9.758	
277†........	"	79.04	"	78.86	65.0	7.855	
278†........	"	47.00	"	47.00	65.0	5.820	
283........	"	22.24	....	....	66.1	3.882	
284........	"	14.08	....	....	67.0	2.908	
285........	"	7.375	....	....	67.0	2.011	
286........	"	4.89	*B–C*	4.86	68.0	1.587	
287........	"	3.57	"	3.58	68.2	1.343	
288........	"	1.71*	"	1.59*	68.4	1.051	
289........	"	0.95	"	0.96	68.6	0.849	
290........	"	0.58	"	0.60	68.8	0.541	
291........	"	0.31	"	0.33	69.0	0.291	
292........	"	0.12	"	0.13	69.0	0.111	
338†........	"	169.9	"	170.7	55.0	11.91	Air, 67°.
339†........	"	130.4	"	131.3	55.5	10.27	
340†........	"	78.4	"	79.1	56.0	7.685	
341†........	"	54.9	"	55.2	56.3	6.261	
510†........	"	118.8	"	120.3	42.8	9.450	
511†........	"	75.1	"	76.1	42.1	7.229	
512†........	"	41.8	"	42.2	45.0	5.201	
513†........	"	177.7	"	180.3	46.5	12.03	
514†........	"	168.6	"	171.7	37 7	11.415	
515†........	"	137.0	"	138.1	37.6	10.101	
516†........	"	110.8	"	112.0	37.3	8.951	
517†........	"	86.2	"	87.2	37.5	7.751	
518†........	"	55.6	"	56.0	37.8	6.015	Air, 68.5°.
519†........	"	27.3	"	27.5	37.9	4.001	

TABLE No. 4—(*Continued*).

BRASS PIPE *VIII.*

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	
57	*C–D*	49.55			69.4°	7.443	For *C–D.*
58	“	40.30			69.4	6.622	
59	“	32.64	*A–B*	33.49	69.4	5.833	
60	“	20.59	“	21.25	69.4	4.501	
61	“	11.96*	“	12.31*	69.4	3.300	
62	“	4.40	“	4.54	69.4	1.850	
63	“	1.38	“	1.50	69.4	0.947	
172	*B–C*	43.96			67.5	6.941	
173	“	29.05			67.5	5.584	
174	“	17.63			69.0	4.117	
175	“	10.51			69.3	3.013	
168†	*A–B*	45.23	*B–D*	44.05	67.5	7.015	For *C–D.*
169†	“	30.81	“	30.09	67.5	5.644	
170†	“	17.58	“	17.49	69.0	4.161	
171†	“	10.82	“	10.58	69.3	3.046	
279†	*C–D*	44.86			66.1	6.956	
280†	“	29.91			65.0	5.513	
281†	“	20.85			65.0	4.439	
282†	“	12.43			65.0	3.288	
379†	“	44.2			53.0	6.691	
394	“	30.66			51.5	5.389	
395	“	22.78			52.2	4.553	
396	“	15.07			52.2	3.584	
397	“	6.18			53.3	2.150	
398	“	6.55			54.6	2.216	
556†	*A–B*	183.1			71	14.97	
557†	“	194.9			37	14.545	
558†	“	160.7			36.7	13.03	
559†	“	134.3			36	11.77	
560†	“	103.4			36	10.15	
561†	“	81.3			36.1	8.791	
562†	“	58.6			36.1	7.370	
563†	“	38.5			36.3	5.770	
564†	“	23.4			36.5	4.332	
565†	“	134.6			39.2	11.87	
566†	“	114.6			48.8	10.99	
567†	“	80.8			48.4	9.063	
568†	“	54.0			49.0	7.252	
569†	“	26.5			49.2	4.824	

BRASS PIPE *IX*.

64	*A–B*	73.36	....	....	69.1°	7.868 (?)	† Heads read in mercury.
65	"	59.15	....	....	69.5	6.777	* Columns vibrating badly.
66	"	39.29	....	....	69.6	5.371-	
67	"	27.24	....	....	69.9	4.356	In Experiment 64 there was evidently a mistake of 5 lbs. in the weight. This was checked by Experiment 70. Velocity should be 7.652.
68	"	15.06	....	....	70.1	3.119	
69	"	6.38	....	....	70.5	1.928	
70	"	73.05	....	....	71.0	7.631	
176†	"	172.6	*B–C*	172.2	67.5	12.28	
177†	"	117.5	"	117.7	67.5	9.884	
178†	"	68.35	"	68.29	69.0	7.286	
179†	"	40.76	"	41.04	69.3	5.334	
275†	"	173.1	"	173.7	66.1	12.31	
276†	"	115.6	"	115.8	65.0	9.758	
277†	"	79.04	"	78.86	65.0	7.855	
278†	"	47.00	"	47.00	65.0	5.820	
283	"	22.24	....	....	66.1	3.882	
284	"	14.08	....	....	67.0	2.903	
285	"	7.375	....	....	67.0	2.011	
286	"	4.89	*B–C*	4.86	68.0	1.587	
287	"	3.57	"	3.53	68.2	1.343	
288	"	1.71*	"	1.59*	68.4	1.051	
289	"	0.95	"	0.96	68.6	0.849	
290	"	0.58	"	0.60	68.8	0.541	
291	"	0.31	"	0.33	69.0	0.291	
292	"	0.12	"	0.13	69.0	0.111	
338†	"	169.9	"	170.7	55.0	11.91	Air, 67°.
339†	"	130.4	"	131.3	55.5	10.27	
340†	"	78.4	"	79.1	56.0	7.685	
341†	"	54.9	"	55.2	56.3	6.261	
510†	"	118.8	"	120.3	42.8	9.450	
511†	"	75.1	"	76.1	42.1	7.229	
512†	"	41.8	"	42.2	45.0	5.201	
513†	"	177.7	"	180.3	46.5	12.03	
514†	"	168.6	"	171.7	37 7	11.415	
515†	"	137.0	"	138.1	37.6	10.101	
516†	"	110.8	"	112.0	37.3	8.951	
517†	"	86.2	"	87.2	37.5	7.751	
518†	"	55.6	"	56.0	37.8	6.015	Air, 68.5°.
519†	"	27.3	"	27.5	37.9	4.001	

TABLE No. 4—(*Continued*).

BRASS PIPE *VIII.*

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
57	*C-D*	49.55			69.4°	7.443 For *C-D*.	†Heads read in mercury.
58	"	40.30			69.4	6.622	*Columns vibrating badly.
59	"	32.64	*A-B*	33.49	69.4	5.833	5.895 Velocities for *A-B*.
60	"	20.59	"	21.25	69.4	4.501	4.549
61	"	11.96*	"	12.31*	69.4	3.300	3.335
62	"	4.40	"	4.54	69.4	1.850	1.870
63	"	1.38	"	1.50	69.4	0.947	0.957
172	*B-C*	43.96			67.5	6.941	
173	"	29.05			67.5	5.584	
174	"	17.63			69.0	4.117	
175	"	10.51			69.3	3.013	
168†	*A-B*	45.23	*B-D*	44.05	67.5	7.015 For *C-D*.	
169†	"	30.81	"	30.09	67.5	5.644	Velocities for *B-D* same as for *B-C*. Experiments 172 to 175.
170†	"	17.58	"	17.49	69.0	4.161	
171†	"	10.82	"	10.58	69.3	3.046	
279†	*C-D*	44.86			66.1	6.956	
280†	"	29.91			65.0	5.513	
281†	"	20.85			65.0	4.439	
282†	"	12.43			65.0	3.288	
379†	"	44.2			53.0	6.691	
394	"	30.66			51.5	5.389	
395	"	22.78			52.2	4.553	
396	"	15.07			52.2	3.584	Air, 67°.
397	"	6.18			53.3	2.150	
398	"	6.55			54.6	2.216	
556†	*A-B*	183.1			71	14.97	
557†	"	194.9			37	14.545	
558†	"	160.7			36.7	13.03	
559†	"	134.3			36	11.77	
560†	"	103.4			36	10.15	
561†	"	81.3			36.1	8.791	Air, 66°.
562†	"	58.6			36.1	7.370	
563†	"	38.5			36.3	5.770	
564†	"	23.4			36.5	4.332	
565†	"	134.6			39.2	11.87	
566†	"	114.6			48.8	10.99	
567†	"	80.8			48.4	9.063	
568†	"	54.0			49.0	7.252	
569†	"	26.5			49.2	4.824	Air, 66°.

BRASS PIPE *XI*.

80	*A-B*	43.99	....	....	69.0°	4.567	†Heads read in mercury.
81	"	29.37	....	....	69.0	3.626	
82	"	18.91	....	....	69.0	2.813	
83	"	11.55	....	....	69.0	2.127	
84	"	4.19	....	....	69.0	1.375	
133†	"	69.44	*B-C*	70.76	67.8	5.953	
134†	"	56.29	"	57.52	67.8	5.271	
135†	"	46.15	"	46.99	67.6	4.704	
136†	"	37.35	"	38.30	69.0	4.182	
137†	"	25.98	"	26.72	69.7	3.409	
138†	"	15.05	"	15.66	70.0	2.517	
139†	"	8.70	"	9.08	70.0	1.832	
372†	"	135.1	"	137.9	69.0	8.741	In 372 and 373 the discharge valve remained in the same position. They show the temperature effect on the discharge very clearly.
373†	"	135.3	"	138.2	56.5	8.497	
374†	"	206.4	"	210.8	57.5	10.872	
375†	"	173.9	"	177.7	58.0	9.854	
376†	"	103.8	"	105.3	55.3	7.261	
377†	"	66.8	"	68.1	55.8	5.651	
378†	"	32.1	"	32.9	56.3	3.726	
479†	"	198.3	"	201.4	37.9	10.077	Pipe *XIII* removed, 372 to 492.
480†	"	158.7	"	161.4	37.8	8.918	Air, 67°.
481†	"	124.2	"	126.2	37.0	7.685	
482†	"	92.3	"	93.9	37.0	6.513	
483†	"	57.9	"	58.8	37.1	4.975	
484†	"	19.1	"	19.4	37.4	2.714	Air, 66°.
485†	"	197.4	"	201.2	47.6	10.386	
486†	"	130.1	"	132.8	49.1	8.238	
487†	"	198.5	"	201.9	39.1	10.170	Air, 69°.
488†	"	198.4	"	201.8	41.1	10.206	
489†	"	199.0	"	202.6	43.9	10.266	
490†	"	283.5	....	....	48.5	12.751	Section *B-C* removed.
491†	"	286.5	....	....	39.9	12.534	
492†	"	233.1	....	....	43.0	11.348	

TABLE No. 4—(*Continued*).

BRASS PIPE *X*.

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
71	*A-B*	48.38	....	....	70.3°	5.311	† Head read in mercury.
72	"	33.20	....	....	70.6	4.278	
73	"	21.19	....	....	70.7	3.301	
74	"	9.28	....	....	71.0	2.056	
75	"	2.59	....	....	71.0	1.107	
147†	"	76.72	*B-C*	76.77	68.3	6.914	
148†	"	59.75	"	59.78	68.0	5.972	
149†	"	40.00	"	40.16	68.0	4.772	
150†	"	29.57	"	29.75	68.0	3.895	
151†	"	17.06	"	16.55	68.1	2.939	
152†	"	11.17	"	11.54	68.3	2.333	
153†	"	3.93	"	3.95	68.7	1.335	
354†	"	51.05	"	51.09	68.1	5.483	
355†	"	50.99	"	50.95	56.6	5.340	
356†	"	78.43	"	78.31	54.0	6.796	
357†	"	40.44	"	40.56	56.2	4.681	
358†	"	15.93	"	16.00	55.2	2.749	
529†	"	181.9	"	181.4	64.8	11.321	
530†	"	148.2	"	147.8	55.2	9.823	
531†	"	114.6	"	114.3	51.0	8.411	
532†	"	180.0	"	178.7	74.1	11.387	
533†	"	139.1	"	137.8	73.4	9.818	
534†	"	173.9	"	173.9	42.5	10.462	
535†	"	145.0	"	145.2	42.0	9.385	
536†	"	110.2	"	110.6	44.0	8.057	
537†	"	74.6	"	75.0	45.2	6.450	
538†	"	171.7	"	171.3	38.0	10.276	
539†	"	127.6	"	127.8	37.1	8.636	
540†	"	92.6	"	93.0	37.2	7.182	
541†	"	56.4	"	56.5	37.2	5.388	
542†	"	32.8	"	32.7	37.1	3.970	
543†	"	149.4	"	149.0	36.7	9.445	

BRASS PIPE *XIII.*

85.	*A–B*	49.61	......	......	69.6°	4.115	† Heads read in mercury.
86.	“	33.48	......	......	69.6	3.294	* Columns vibrating badly; vibrations greater in 293 than in 294; critical-velocity region.
87.	“	17.21	......	......	69.6	2.250	
88.	“	5.60	......	......	69.6	1.332	
140†	“	217.4	*B–C*	218.1	67.8	9.635	
141†	“	176.6	“	177.2	67.8	8.531	
142†	“	144.3	“	144.8	68.6	7.615	
143†	“	117.5	“	117.5	69.0	6.658	
144†	“	82.0	“	82.4	69.7	5.426	
145†	“	48.3	“	48.6	70.0	4.007	
146†	“	27.9	“	28.0	70.0	2.917	
293.	“	7.11*	“	7.99*	60.6	1.765	
294.	“	5.99*	“	6.56*	61.0	1.215	
295.	“	4.47	“	4.47	61.5	1.270	
296.	“	3.13	“	3.15	62.0	0.898	
297.	“	1.85	“	1.87	62.1	0.540	
298.	“	0.91	“	0.84	62.4	0.267	
319†	“	220.3	“	220.3	65.2	9.578	
320†	“	143.2	“	144.1	65.2	7.507	
321†	“	76.6	“	77.3	65.2	5.231	
330†	“	137.6	“	138.4	56.5	7.229	
331†	“	83.8	“	84.5	56.5	5.418	Air, 70°.
332†	“	192.5	“	193.5	56.5	8.775	
493†	“	406.7	......	......	63.2	13.651	
494†	“	300.3	......	......	60.8	11.43	
495†	“	408.3	......	......	46.2	13.20	Air, 67°. Removed *XI*, except entry length and piezometer *A*. End length of *XI*, with reducer, screwed into *A*. *XIII* *B–C* removed.
496†	“	353.7	......	......	48.6	12.22	
497†	“	282.4	......	......	47.4	10.71	
498†	“	213.2	......	......	46.8	9.097	
499†	“	209.2	......	......	38.2	8 777	
500†	“	302.5	......	......	36.3	10.780	
501†	“	402.5	......	......	37.4	12.78	
502†	“	252.9	*B–C*	253.6	50.0	10.07	*B–C* put on.
503†	“	199.7	“	201.2	50.0	8.836	
504†	“	124.3	“	125.1	47.0	6.664	
505†	“	81.7	“	82.0	46.0	5.227	
506†	“	26.4	“	27.2	39.0	2.744	
507†	“	64.0	“	63.9	38.8	4.443	Air, 65°.
508†	“	112.7	“	114.0	38.5	6.182	
509†	“	155.9	“	158.4	38.9	7.468	

TABLE No. 4—(*Continued*).

BRASS PIPE *XII*.

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.		Temp. Fahr.	Velocity, in feet per second.				Remarks.
76	*A–B*	55.18	....	....		72.0°	4.940				† Heads read in mercury.
77	"	39.17	....	....		72.3	4.002				
78	"	21.32	....	....		72.4	2.865				
79	"	9.53	....	....		72.5	1.809				
154†	"	205.9	*B–D*	187.8	For *C–D*	68.3	10.477	For *A–B*	10.165	For *B–D*	
155†	"	160.4	"	146.3		68.0	9.050		8.780		
156†	"	107.7	"	98.64		68.0	7.232		7.016		
157†	"	80.03	"	73.15		68.0	5.903		5.727		
158†	"	46.63	"	42.56		68.1	4.454		4.321		
159†	"	31.16	"	28.41		68.3	3.536		3.430		
160†	"	11.74	"	10.47		68.7	2.023		1.963		
161	....	....	*B–C*	185.6		68.3	10.165				Experiments 161 to 167 were simultaneous with 154 to 160. The heads were read on a water gauge. The heads given for *B–D* (154 to 160) are really for *C–D*, the *B–C* heads (161 to 167) having been subtracted.
162	....	....	"	144.8		68.0	8.780				
163	....	....	"	97.4		68.0	7.016				
164	....	....	"	72.32		68.0	5.727				
165	....	....	"	42.27		68.1	4.321				
166	....	....	"	28.15		68.3	3.430				
167	....	....	"	10.52		68.7	1.963				
267†	*A–B*	135.4	*C–D*	124.2		68.8	8.270	*A-B*	8.024	*C-D*	
268†	"	73.22	"	67.4		69.2	5.817		5.644		
269†	"	133.0	"	122.2		65.0	8.137	*A–B*	7.895	*C–D*	
270†	"	104.4	"	95.94		65.0	7.088		6.877		
271†	"	64.35	"	59.34		65.0	5.362		5.203		
272†	"	48.35	"	44.59		65.0	4.556		4.422		
273†	"	28.92	"	26.61		65.0	3.390		3.290		
274†	"	207.2	"	190.2		65.3	10.510		10.197		
352†	....	....	"	125.2		68.1			8.061	*C–D*	
353†	....	....	"	125.0		56.6			7.850		Air, 68°.
359†	....	....	"	192.6		54.0			10.004		
360†	....	....	"	156.7		55.1			8.910		
361†	....	....	"	82.8		55.0			6.172		Air, 67°.
362†	....	....	"	34.6		56.0			3.738		
520†	*A–B*	202.7	"	186.1		45.4	9.907	*A–B*	9.612	*C–D*	
521†	"	145.8	"	133.4		45.4	8.170		7.927		
522†	"	81.2	"	74.2		45.7	5.843		5.669		
523†	"	258.7	....	....		54.9	12.425				*B–D* removed.
524†	"	249.5	....	....		53.0	11.875				Air, 71.5°.
525†	"	235.1	....	....		39.0	10.609				
526†	"	288.1	....	....		39.4	11.910				
527†	"	243.5	....	....		43.0	10.950				
528†	"	290.0	....	....		45.6	12.175				

Brass Pipe *XV*.

94....................	*A-B*	44.58	....	....	70.6°	2.892	†Heads read in mercury.
95....................	“	33.83	....	....	70.6	2.596	
96....................	“	19.26	....	....	70.6	2.362	
97....................	“	8.623	....	....	70.0	1.248	
98....................	“	2.001	....	....	70.0	0.300	
117†..................	“	180.2	*B-C*	194.2	69.5	6.761	
118†..................	“	155.4	“	167.6	70.0	5.946	
119†..................	“	129.6	“	139.5	70.0	5.384	
120†..................	“	99.9	“	107.6	70.2	4.630	
121†..................	“	67.84	“	72.47	70.4	3.730	
122†..................	“	13.60	“	20.72	70.9	1.953	
123†..................	“	275.8	“	299.3	70.6	8.304	
124†..................	“	218.4	“	235.6	71.0	7.249	
222†..................	“	510.5	....	....	74.8	11.868	*B-C* removed.
223†..................	“	411.4	....	....	75.5	10.519	
224†..................	“	319.1	....	....	76.0	9.102	
225†..................	“	207.7	....	....	75.7	7.109	
226†..................	“	114.2	....	....	75.1	5.075	
227†..................	“	45.32	....	....	73.9	3.009	
228†..................	“	29.05	....	....	73.1	2.394	
229†..................	“	18.16	....	....	73.0	1.952	
230†..................	“	9.33	....	....	73.1	1.362	
237†..................	“	278.4	*B-C*	303.3	72.9	8.398	
238†..................	“	179.2	“	194.5	70.9	6.537	
239†..................	“	122.7	“	133.7	73.6	5.277	
240†..................	“	273.4	“	298.4	70.7	8.217	Pipes Nos. 2 and 3 interchanged, placing Section *B-C* up stream.
241†..................	“	233.5	“	255.4	70.8	7.575	
242†..................	“	151.4	“	164.6	71.0	5.902	
253†..................	“	281.9	“	284.1	70.2	8.489	Pipes cleaned. *A-B* up stream.
254†..................	“	227.5	“	228.0	70.3	7.500	
255†..................	“	145.4	“	146.2	70.4	5.852	
256†..................	“	52.55	“	52.66	70.7	3.276	
257†..................	“	32.58	“	33.84	71.7	2.537	
258†..................	“	11.41	“	11.53	72.9	1.767	
368†..................	“	174.4	“	175.2	53.7	6.257	
369†..................	“	242.0	“	243.0	55.0	7.578	
370†..................	“	292.2	“	293.4	55.4	8.460	
371†..................	“	102.2	“	102.6	56.0	4.682	
413†..................	“	270.1	“	271.0	46.0	7.800	
414†..................	“	176.2	“	177.1	47.5	6.145	
415†..................	“	175.7	“	176.8	47.0	6.188	Air, 68°.
416†..................	“	109.8	“	110.4	49.6	4.781	
417†..................	“	52.3	“	53.0	45.7	3.183	Air, 69.5°.

TABLE No. 4—(*Continued*).

Brass Pipe *XIV*.

No. of Exp't.	Section.	Loss of head, in water. Feet per 100.	Section.	Loss of head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
246†	*A-B*	304.0	*B-C*	308.8	70.2°	9.957	Note: Experiments 89 to 93, 125 to 132, 231 to 236, were rejected. The results on the two sections were not consistent, and when the pipe was taken down the ends were found to be jammed. (This pipe is very thin.) The jammed ends were cut off, and the pipes were refaced and rethreaded. Subsequent examinations showed no trouble.
247†	"	236.3	"	240.6	70.3	8.618	
248†	"	154.1	"	157.1	70.4	6.685	
249†	"	97.16	"	98.73	70.7	5.179	
250†	"	54.49	"	55.05	71.7	3.722	
251†	"	28.92	"	29.28	72.9	2.608	
252†	"	12.49	"	13.80	73.5	1.743	
363†	"	203.0	"	206.4	54.8	7.661	
364†	"	151.7	"	154.5	56.0	6.483	
365†	"	89.6	"	91.5	56.4	4.813	
366†	"	39.0	"	39.8	56.8	2.943	Air, 67.5°.
367†	"	310.5	"	317.1	54.1	9.760	
447†	"	202.0	"	204.7	47.2	7.427	Air, 76°.
448†	"	163.3	"	166.0	44.4	6.548	
449†	"	134.4	"	136.4	45.8	5.870	Air, 68.5°.
450†	"	96.4	"	97.9	42.4	4.859	
451†	"	61.5	"	62.2	42.8	3.780	
452†	"	35.7	"	36.6	42.2	2.825	Air, 66.5°.
453†	"	302.3	"	306.8	42.2	9.314	
454†	"	256.4	"	260.2	49.4	8.584	
455†	"	571.3	......	......	61.4	14.011	
456†	"	479.7	......	......	59.8	12.659	Air, 72°.
457†	"	373.1	......	......	59.8	10.936	
458†	"	266.8	......	......	60.8	9.047	
459†	"	265.3	......	......	42.5	8.625	Air, 71°.
460†	"	359.0	......	......	45.0	10.333	
461†	"	442.6	......	......	47.9	11.772	
462†	"	565.7	......	......	48.8	13.566	Air, 74°.
463†	"	555.7	......	......	37.8	13.036	Air, 68°.
464†	"	517.3	......	......	38.8	12.567	
465†	"	429.5	......	......	38.0	11.270	
466‡	"	324.4	......	......	37.5	9.569	Air, 65.8°.
467†	"	216.5	......	......	37.6	7.553	
468†	"	93.3	......	......	37.6	4.792	Air, 65°.

† Heads read in mercury.

BRASS PIPE *XVI.*

99....................	*A-B*	44.46	....	....	67.7°	2.676	† Heads read in mercury. * Sudden change in gauge columns, probably due to the settling of the valve.
100....................	“	28.59	....	....	68.0	2.074	
101....................	“	18.29	....	....	69.0	1.378	
102....................	“	7.804	....	....	69.5	0.602	
103†....................	“	178.6	*B-C*	178.4	70	5.135	
104†....................	“	136.7	“	136.6	70.2	4.424	
105†....................	“	105.4	“	105.2	70.9	3.840	
106†....................	“	76.4	“	76.6	71.0	3.230	
107†....................	“	48.29	“	48.6	71.2	2.721	
108†....................	“	24.15	“	23.0	71.9	1.790	
109†....................	“	60.61	“	62.36	69.5	2.917	
110†....................	“	56.53	“	58.49	70.0	2.866	
111†....................	“	37.36	“	38.42	70.0	2.559	
112†....................	“	31.94	“	31.93	70.2	2.309	
113†....................	“	11.72	“	11.71	70.4	0.911	
114†....................	“	5.86	“	5.97	70.9	0.451	
115†....................	“	271.4	“	276.6	70.6	6.542	
116†....................	“	229.9	“	234.7	71.0	5.993	
213†....................	“	512.8	....	....	74.5	9.448	*B-C* removed.
214†....................	“	423.0	....	....	75.0	8.467	
215†....................	“	323.9	....	....	75.1	7.277	
216†....................	“	231.1	....	....	75.2	6.001	
217†....................	“	124.6	....	....	75.3	4.255	
218†....................	“	47.25	....	....	75.1	2.620	
219†....................	“	32.24	....	....	75.1	2·282	
220†....................	“	19.89	....	....	75.1	1.614	
221†....................	“	8.97	....	....	75.2	0.730	
243†....................	“	276.6	*B-C*	280.5	70.7	6.606	Pipes Nos. 2 and 3 interchanged, placing section *B-C* up stream.
244†....................	“	227.0	“	230.2	70.8	5.816	
245†....................	“	136.2	“	137.9	71.0	4.437	
314†....................	“	178.0	“	180.7	64.3	5.107	Section *A-B* up stream.
315†....................	“	129.3	“	131.6	64.5	4.298	
316†....................	“	76.47	“	80.14	64.8	3.248	
317†....................	“	39.12	“	37.27	65.0	2.523	
318†....................	“	284.2	“	288.2	65.0	6.645	
322†....................	“	521.1	“	....	64.0	9.337	*B-C* removed.
323†....................	“	446.1	....	....	64.0	8.552	
324†....................	“	324.5	....	....	64.0	7.143	
333†....................	“	496.8	....	....	55.0	8.940	
334†....................	“	388.2	....	....	55.8	7.809	

TABLE No. 4—(*Continued*).

Brass Pipe *XV*—(*Continued*).

No. of Exp't.	Section.	Loss of Head, in water. Feet per 100.	Section.	Loss of Head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
418†	*A-B*	287.8			44.1	8.168	
419†	"	443.9			43.2	9.991	No. 419 seems erratic. No. 443 taken as a check.
420†	"	364.3*			46.4	9.254	
421†	"	440.7			38.2	10.237	*In No. 420 the head decreased by jumps, probably due to the valve settling slightly.
422†	"	374.3			38.0	9.327	
423†	"	314.1			37.7	8.450	
424†	"	255.7			37.6	7.522	Air, 69°.
425†	"	194.3			37.6	6.444	
426†	"	132.6			37.5	5.194	Air, 66°.
427†	"	71.4			37.6	3.752	
443†	"	440			43.7	10.344	
444†	"	461.4			45.8	10.695	
445†	"	408.7			46.2	9.965	
446†	"	325.9			46.1	8.759	Air, 71°.

TABLE No. 4—(*Continued*).

BRASS PIPE *XVI*—(*Continued*).

No. of Exp't.	Section.	Loss of Head, in water. Feet per 100.	Section.	Loss of Head, in water. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
335†	*A-B*	267.0*	....	....	56.5	6.327	Air, 62°.
336†	"	250.8	....	....	57.0	6.130	
337†	"	110.1	....	....	57.4	3.870	
428†	"	496.8	....	....	43.5	8.746	
429†	"	409.3	....	....	45.0	7.869	
430†	"	307.8	....	....	47.8	6.754	Air, 78°.
431†	"	307.4	....	....	39.4	6.659	
432†	"	389.9	....	....	38	7.562	
433†	"	436.2	....	....	38.3	8.047	
434†	"	481.0	....	....	38.6	8.516	
435†	"	267.9	*B-C*	271.1	39.0	6.140	Air, 65°.
436†	"	213.5	"	216.6	39.0	5.424	
437†	"	158.7	"	162.6	38.6	4.751	
438†	"	103.2	"	101.3	38.5	4.071	Air, 68°.
439†	"	66.7	"	64.9	38.5	3.142	
440†	"	33.7	"	33.0	38.5	1.635	
441†	"	19.1	"	18.8	38.4	0.958	
442†	"	7.2	"	7.2	38.3	0.387	

TABLE No. 5.—GALVANIZED PIPE *XVII.*

No. of Exp't.	Loss of head. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
592	70.89	35.5°	10.036	All heads measured in mercury.
593	39.90	35.8	7.498	
594	54.00	44.4	8.762	
595	39.59	42.5	7.452	Air, 64°.
596	25.85	41.2	6.008	
597	13.17	40.9	4.229	
598	5.88	41.2	2.795	
599	62.46	35.5	9.374	
600	55.35	35.6	8.476	
601	28.75	35.6	6.340	
602	16.51	36.4	4.766	
620	44.19	59.5	7.955	
621	26.21	51.2	6.092	
741	44.87	79	8.375	Experimental pipe reversed (Piezometer *B* up stream) for these experiments.
742	43.89	43.6	7.813	
743	43.67	35.6	7.711	
744	74.36	35.4	10.155	
745	73.74	35.2	10.118	
746	55.61	35.5	8.860	
747	26.82	35.2	6.041	
748	13.21	35.4	4.216	
749	7.37	35.6	3.111	

GALVANIZED PIPE *XVIII.*

631	51.61	44.6°	9.469	All heads measured in mercury.
632	41.11	42.9	8.321	
633	30.06	42.4	7.060	
634	20.60	42.0	5.708	
635	10.40	42.5	3.913	
636	4.99	42.8	2.618	
637	75.83	36.0	11.470	
638	64.87	36.0	10.554	
639	50.87	71.5	9.635	
640	32.41	64	7.548	
641	74.37	39.8	11.427	
642	74.89	35.8	11.405	
643	50.10	36.1	9.159	
644	36.74	36.2	7.757	
645	19.99	35.8	5.513	
646	8.47	36.5	3.423	
737	51.74	37	9.271	Experimental pipe reversed (Piezometer *B* up stream) for these experiments.
738	31.00	37.2	7.047	
739	15.52	38.2	4.811	
740	71.22	35.1	10.916	

TABLE No. 5—*Continued.*—GALVANIZED PIPE *XIX.*

No. of Exp't.	Loss of head. Feet per 100.	Temp. Fahr.	Velocity, in feet per second.	Remarks.
622	96.62	55.0°	10.223	All heads measured in mercury.
623	61 20	49	7.909	
624	33.62	48	5.747	
625	16.72	48.2	3.894	
626	7.36	49.2	2.460	
627	143.31	77	12.783	
628	144.72	37.5	12.339	
629	80.22	37.5	9.049	
630	37.70	37.7	5.996	
755	98.04	78	9.994*	Experimental pipe reversed for these experiments. * Error in time? Velocity may be 10.316. See Experiment 768.
756	97.56	50	10.079	
757	97.70	50.7	10.248	
758	78.17	52.0	9.028	
759	78.13	49.0	9.011	
760	39.71	48.5	6.267	
761	22.81	49.0	4.610	
762	148.47	48.6	12.560	
763	149.69	37.1	12.578	
768	98.16	77.2	10.298	
769	77.00	80.2	9.196	
770	77.88	45.9	8.931	
771	97.65	45.8	10.062	

GALVANIZED PIPE *XX.*

603	166.6	61.5	10.885	All heads measured in mercury.
604	109.1	58.8	8.725	
605	165.6	37.2	10.653	
606	149.6	36.8	10.145	
607	116.5	35.9	8.877	
608	79.15	35.5	7.240	
609	46.20	35.6	5.499	
610	21.12	35.8	3.634	
611	10.34	39.0	2.487	
612	5.03	40.6	1.669	
613	113.3	42.6	8.781	
614	91.35	49.5	7.877	
615	112.6	47.0	8.775	
616	59.93	46.4	6.308	
617	112.2	77.2	8.896	
618	51.56	78.3	5.965	
619	164.5	79.2	10.884	
764	166.4	48.4	10.744	Experimental pipe reversed for these experiments.
765	108.2	45.2	8.607	
766	57.4	45.2	6.156	
767	25.9	52	4.077	

TABLE No. 5—*Continued.*—GALVANIZED PIPE *XXI.*

No. of Exp't.	Loss of head. Feet per 100.	Temp. Fahr.	Velocity in feet per second.	Remarks.
570	408.5	71.5°	11.411	All heads measured in mercury.
571	205.1	71.5	8.009	
572	341.5	68	10.394	
573	257.2	54	8.956	
574	181.3	52.2	7.489	
575	125.6	59.2	6.219	
576	84.71	57.8	5.085	
577	53.77	57.1	4.039	
578	31.68	59.8	3.067	
579	374.9	38.5	10.763	
580	311.8	38.6	9.830	
581	258.4	37.4	8.938	
582	195.3	37.4	7.739	
583	133.8	37.5	6.394	
584	84.05	38.0	5.025	
585	47.50	38.4	3.756	
586	5.97	42.6	1.305	
587	11.94	44.0	1.856	
588	25.69	47.2	2.744	
589	268.1	46.3	9.112	
590	142.6	47.8	6.621	
591	388.3	49.7	10.992	
750	316.5	45.1	9.795	Experimental pipe reversed (Piezometer *B* up stream) for these experiments.
751	315.4	36.4	9.651	
752	244.5	38.9	8.464	
753	161.7	36.4	6.834	
754	77.7	39.0	4.731	

REDUCTION TO STANDARD TEMPERATURES.

Brass Pipes.—After a few experiments had been made on each pipe, a preliminary logarithmic plotting of the results showed that for each of the fifteen brass pipes the loss of head varied very nearly as the 1.75 power of the velocity. Advantage was taken of this fact to bring the results to chosen standard temperatures. Plates XXXVIII and XXXIX were drawn, using heads in feet per 100 ft. as ordinates and the values of $V^{1.75}$ as abscissas. For any particular pipe on these plates, it will be noticed that there is a gradual increase in the ordinate as the temperature of the water drops, and that points of about the same temperature lie approximately on a straight line. Such lines were drawn for standard temperatures of 70, 55 and 40° Fahr. The difference in ordinates between two lines, divided by the ordinate, gives the ratio or percentage of increase or decrease in the loss of head for a change of temperature of 15 degrees. The percentage is always less for 40° than for 55°, as the ordinate itself is greater. In those cases in which the distances between the lines were different, thus resulting in two different percentages at 55°, the average of these two percentages was taken. The three percentages reduced centages per 10°, instead of 15°, were then plotted as ordinat temperatures as abscissas, and a smooth curve was drawn thro three points. The percentage correction per 10° for any o temperature was read from these curves. The diagram used is in Fig. 3, in which it will be seen that for all pipes the correctio is about 4 per cent. Another diagram was therefore constructe which the 4% corrections of the different heads for any temp interval could be read, and the correction desired was then o by the slide-rule.

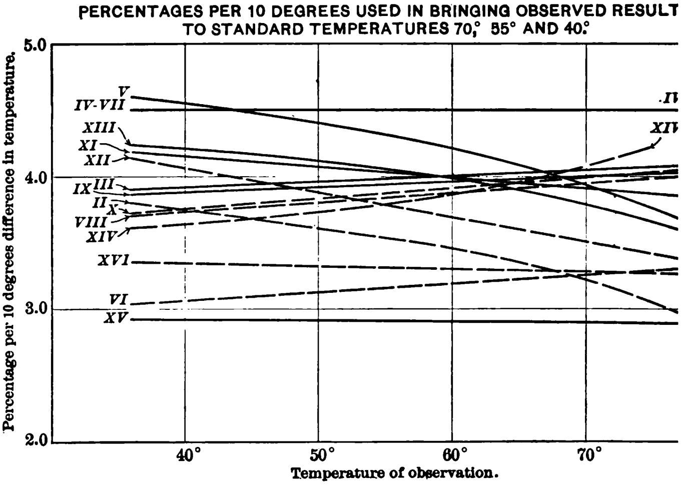

FIG. 3.

In Fig. 3 it will be noticed that for some pipes the per curves slope downward for increasing temperatures, and fo pipes there is an opposite tendency. While it is not to be mai that these corrections are accurate, it is believed that they are within several tenths of 1 per cent. If this is true, a profile Fig. 3, at 70°, 55° or 40°, made by taking the percentages and p them with respect to the pipe diameters, gives indications of a relation between the temperature effects and the diameters evidence, however, is not conclusive enough to justify the sta that this is a fact.

Aside from errors in judgment in drawing the lines on XXXVIII and XXXIX, there is at least one way in which small

PLATE XXXVIII.
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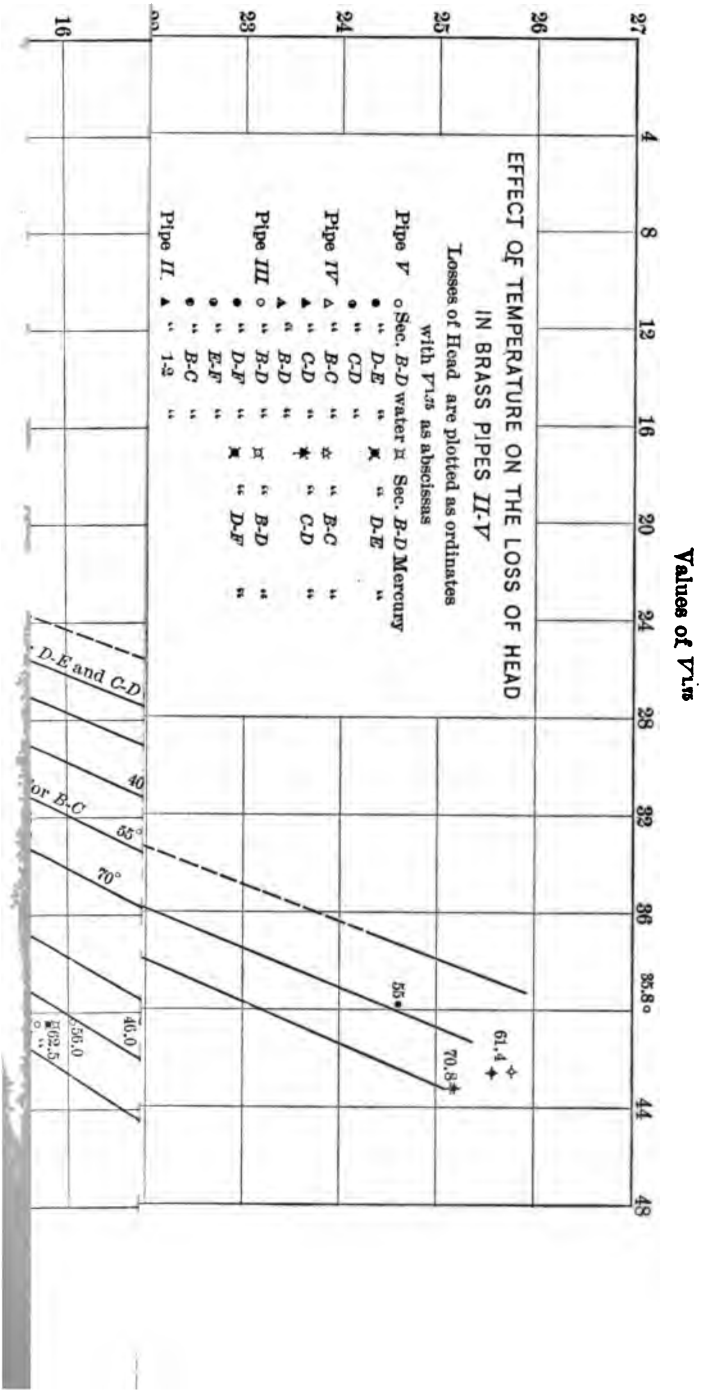
MAY, 1903.
SAPH AND SCHODER ON
FLOW OF WATER IN PIPES.



EFFECT OF TEMPERATURE ON THE LOSS OF HEAD IN BRASS PIPES II-V

Losses of Head are plotted as ordinates with $V^{1.75}$ as abscissas two percentages was taken. The three percentages reduced to pe centages per 10°, instead of 15°, were then plotted as ordinates, wit temperatures as abscissas, and a smooth curve was drawn through th three points. The percentage correction per 10° for any observe temperature was read from these curves. The diagram used is show in Fig. 3, in which it will be seen that for all pipes the correction facto is about 4 per cent. Another diagram was therefore constructed, fro which the 4% corrections of the different heads for any temperatu interval could be read, and the correction desired was then obtaine by the slide-rule.

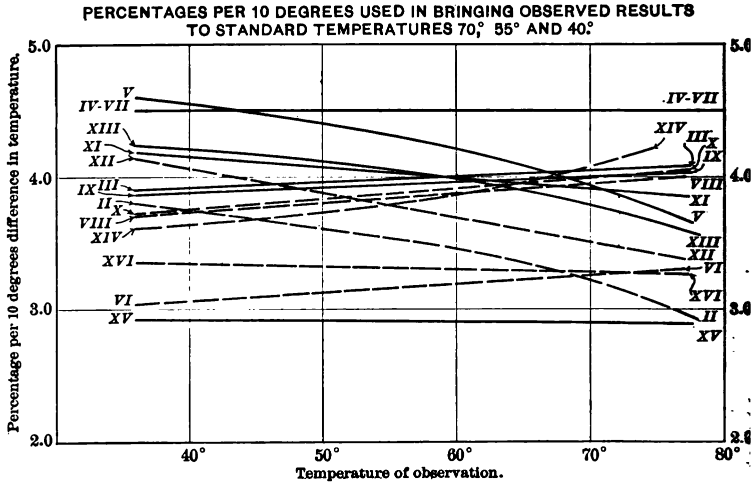


FIG. 3.

In Fig. 3 it will be noticed that for some pipes the percentag curves slope downward for increasing temperatures, and for othe pipes there is an opposite tendency. While it is not to be maintaine that these corrections are accurate, it is believed that they are corre within several tenths of 1 per cent. If this is true, a profile view Fig. 3, at 70°, 55° or 40°, made by taking the percentages and plottin them with respect to the pipe diameters, gives indications of a sinuo relation between the temperature effects and the diameters. Th evidence, however, is not conclusive enough to justify the statem that this is a fact.

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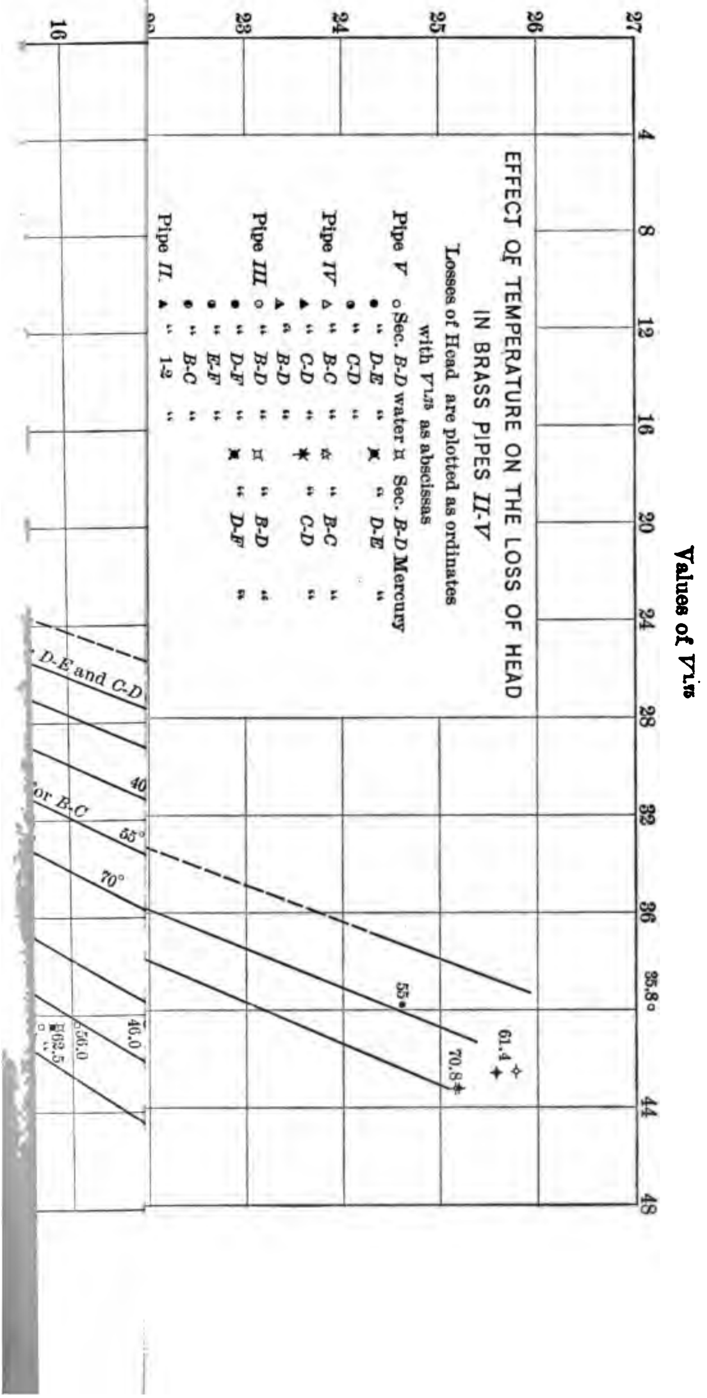

for 10° difference in temperature, showed themselves to be in error. More consistent reduction factors were obtained by using large-scale logarithmic paper for a preliminary plotting. Upon Pipes *XVII* and *XXI* very little evidence of temperature effect could be found, and the results were not corrected. For Pipe *XIX* 1.5% for 10° was found to apply; and for Pipe *XX* 1.25% was used. For Pipe *XVIII* the results were changed by 5% in the neighborhood of 40°, but, from all appearances, this would be too large in the vicinity of 70 degrees. Pipe *XVIII*, however, seems to be exceptional, in several respects.

Logarithmic Plotting of Results.—After being brought to the standard temperatures the results were plotted on regular, 10-in. base, logarithmic, cross-section paper,* plotting losses of head as ordinates and velocities as abscissas. The use of such paper is now so common that no explanation is considered necessary.

This plotting is shown in Plate XL, which contains the results for both brass and galvanized pipes, plotted, however, for different origins. For the brass pipes, lines are drawn for the three standard temperatures, but for only one experimental section on each pipe, in order to avoid confusion.

Close inspection will show that in some instances the points do not lie exactly on the line. In such cases investigation was made to see if the reason existed in an error in the temperature reduction factor, but it was found that the differences were due to other causes. Sometimes the results obtained by mercury readings were slightly different from those taken in water. Altogether, however, the lines fit the points very well. Attention may be called here to the great uniformity in slope for the brass pipes.

Table No. 6 gives the values of m and n, as determined from the logarithmic plotting. In cases, however, where the points do not lie exactly on a straight line, or where, as in Pipe *XVI* at 40°, the points which determine the line are few in number, and for high velocities, there is usually a little uncertainty in drawing the line. In this way, small variations in m may be brought about, but when it is considered that there is a corresponding change in n, it will be seen that the two values taken together will represent the conditions quite accurately.

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* The writers were supplied with this paper by John R. Freeman, M. Am. Soc. C. E. whose generosity and kindly interest they desire to acknowledge.

PLATE XXXIX.
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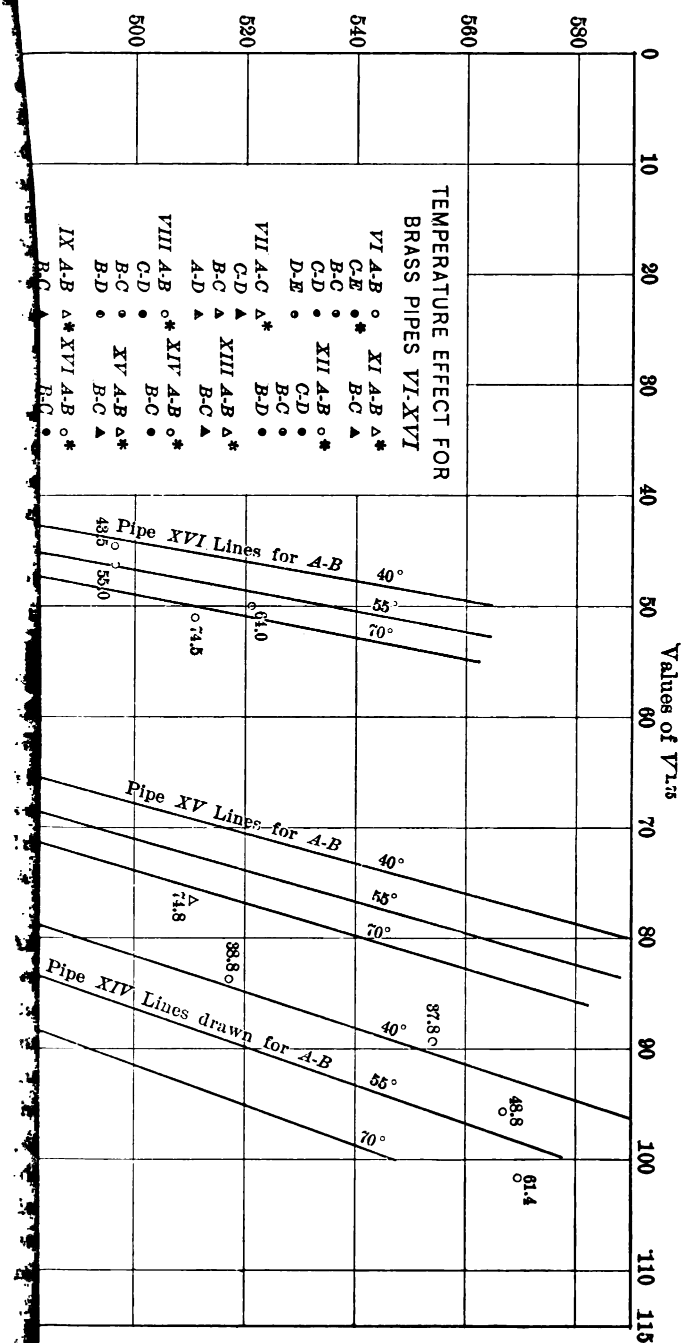
MAY, 1908.
SAPH AND SCHODER ON
FLOW OF WATER IN PIPES.



In Table No. 6 there will also be found the values of *m*, as taken from Plates XXXVIII and XXXIX on which the points were plotted with heads as ordinates and the values of $V^{1.75}$ as abscissas. The slopes of the lines drawn must give the values of *m* corresponding to the power, 1.75, of the velocity. Upon comparing these values of *m* with those obtained from the logarithmic plotting, it is seen that they check quite well. These values of *m* are greater or less than the values determined logarithmically, accordingly as 1.75 is greater or less than the true *n*. Taken altogether, with these values of *m*, and $n = 1.75$, we can represent closely the whole series of experiments.

The method of least squares could also be applied in getting the values of *m* and *n*. This was tried in the case of Pipe *II*, and gave a close check upon the values taken from the logarithmic sheet. The method of least squares, however, is not as convenient, and does not expose the errors which may occur.

The differences in results for two sections of the same pipe do not seem to be accounted for through differences in diameter, except in Pipes *VI*, *VIII* and *XII*, where differences were known to exist and were allowed for in the calculations. In the other pipes small differences in resistance are noted, but must be accounted for otherwise. As first used, the two experimental sections of Pipe *XV* showed very discordant results, as the table and the logarithmic plotting show. After being cleaned by drawing waste cotton through, the results were more consistent. During the cleaning a small scale of brass was drawn out, and it seems very probable that the small differences noted in other pipes may be due to similar causes.

Table No. 7 gives the values of *m* and *n* for the galvanized pipes. As shown by the logarithmic plotting, Pipe *XVIII* seems to be an exceptional pipe. Even though of smaller diameter than Pipe *XVII* the losses of head, for velocities above 2.4 ft. per second, are lower than in Pipe *XVII* for the same velocity. It will be remembered, also, that Pipe *XVIII* is the one which showed high temperature effects in the neighborhood of 40°, and that it is the only one of the galvanized pipes which had been in use before the experiments. A slight, silt-like deposit which had occurred on the inner walls was entirely insufficient to relieve the roughness. Although the galvanized pipes do not seem to give such consistent results as the brass pipes, the experiments were made with the same care.

		40	3.90	1.731	3.74		40	3.72	1.738	
	B-C	70	3.25	1.739	....					
	B-D	70	3.33	1.736	....					
XIII	*A-B*	70	4.25	1.733	4.11	*B-C*	70	4.25	1.733	
		55	4.45	1.730	4.32		55	4.45	1.730	
		40	4.70	1.743	4.61		40	4.70	1.743	
XIV	*A-B*	70	5.52	1.745	5.46	*B-C*	70	5.52	1.755	
		55	5.80	1.747	5.78		55	5.95	1.740	
		40	6.28	1.742	6.13		40	6.45	1.731	
XV	*A-B*	70	6.36	1.774	6.76	*A-B*	70	6.66	1.764 }	*A-B* and *B-C* in last columns before cleaning.
		55	6.71	1.768	7.05	*B-C*	70	7.12	1.765 }	
		40	7.07	1.771	7.36					
	B-C	70	6.36	1.774	....					
		55	6.71	1.768	....					
		40	7.07	1.771	....					
XVI	*A-B*	70	9.78	1.771	10.21	*B-C*	70	9.23	1.785	
		55	10.13	1.780	10.69		55	9.88	1.794	
		40	10.28	1.795	11.25					

TABLE No. 6—BRASS PIPES *II* TO *XVI*, INCLUSIVE.

VALUES OF *m* AND *n*. (*m* FOR HEAD IN FEET PER 100.)

Pipe.	Section.	Temp. Fahr.	*m*.	*n*.	*m* from $V^{1.75}$ diagrams.	Section.	Temp. Fahr.	*m*.	*n*.	
II		70°	0.252	1.749	0.253					
		55	0.267	1.748	0.268					
		40	0.282	1.750	0.282					
III	*B-D*	70	0.372	1.754	0.377	*D-F*	70°	0.375	1.750	
		55	0.386	1.767	0.400	*E-F*	70	0.357	1.776	Few points for *E-F*.
		40	0.420	1.755	0.425					
IV	*B-C*	70	0.485	1.744	0.472	*C-D*	70	0.497	1.730	
		55	0.507	1.753	0.503		55	0.525	1.741	
		40	0.545	1.744	0.550					
V	*D-E*	70	0.584	1.747	0.580	*B-D*	70	0.584	1.747	
		55	0.600	1.763	0.613	*C-D*	40	0.660	1.755	
		40	0.660	1.755	0.660					
VI	*C-E*	70	0.770	1.737	0.788	*D-E*	70	0.729	1.755	
		40	0.880	1.738	0.864	*C-D*	70	0.850	1.722	
	A-B	55	0.850	1.738	0.822	*B-C*	70	0.815	1.758	
		40	0.895	1.743	....					
VII	*A-C*	55	1.165	1.753	1.172					
		40	1.340	1.716	1.100	*B-C*				
	C-D	55	1.170	1.748	....	*A-C*	70	1.11	1.744	All at 70° were made before pipe was set up the second time.
		40	1.340	1.709	....	*C-D*				
VIII	*A-B*	70	1.53	1.732	1.615	*C-D*	70	1.51	1.737	$V^{1.75}$ diagram for *C-D VII* gives $m = 1.47$ and 1.55.
		55	1.630	1.756	1.675		55	1.64	1.730	
		40	1.74	1.757	1.775					
IX	*A-B*	70	2.04	1.763	2.10	*B-C*	70	2.05	1.760	
		55	2.20	1.751	2.32		55	2.22	1.749	
		40	2.42	1.740	2.36		40	2.40	1.750	
X	*A-B*	70	2.60	1.746	2.58	*B-C*	70	2.60	1.746	
		55	2.72	1.753	2.73		55	2.75	1.745	
		40	2.94	1.741	2.90		40	2.94	1.741	
XI	*A-B*	70	3.03	1.746	3.02	*B-C*	70	3.12	1.744	
		55	3.22	1.747	3.21		55	3.26	1.749	
		40	3.45	1.743	3.41		40	3.51	1.747	
XII	*A-B*	70	3.39	1.744	3.34	*C-D*	70	3.27	1.739	
		55	3.61	1.739	3.52		55	3.48	1.738	

TABLE No. 7.

GALVANIZED PIPES *XVII* TO *XXI*, INCLUSIVE. VALUES OF m AND n (m FOR HEADS IN FEET PER 100).

Pipe.	Temp. Fahr.	m.	n.
XVII	....	0.820	1.931
XVIII	40°	0.905	1.802
XIX	40	1.320	1.863
XX	70	1.74	1.906
	40	1.81	1.910
XXI	....	3.52	1.961

CRITICAL VELOCITY.

One of the most interesting features of Plate *XL*, Fig. 1, is the evidence regarding critical velocity. As might be expected, this phenomenon is most prominent on Pipe *XVI*, where the peculiar effects are evident at relatively high velocities. For low velocities, up to 2.5 ft. per second, the slope is 1, showing that the loss of head is directly proportional to the velocity. Above a velocity of 3.5 ft. the slope is about 1.78. Between these velocities there is a region where the slope gradually changes and reaches a value considerably greater than 2. The cause, as Osborne Reynolds, F. R. S., has shown* is a change in the conditions of flow.

The effect of temperature on the point at which the change takes place is well shown by Pipe *XVI*, and also in Fig. 4, which has been prepared from the observations of Hagen.† For these experiments $0.5 \frac{V^2}{2g}$ was allowed for head lost at entrance (square-edged). This may not be quite correct. The pipes were short, being 1.55, 3.57 and 3.42 ft. long, for the small, medium and larger sized pipes, respectively. Any other allowance for entry would not change the relative position of the lines to any great extent. The figure has been introduced to show the effect of temperature in the critical-velocity region, particularly for greater temperatures than the writers could obtain, and also to verify the indications of the present experiments. The principal point to be brought out is that the change takes place at higher velocities for lower temperatures.‡ The change to the power

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* *Proceedings*, Royal Society, Vol. 35, 1883.

† "Abhandlungen der Berliner Akademie," 1854.

‡ Some recent experiments substantiating this very point came to the writer's notice after this paper had been written. They are by Messrs. E. J. Coker and S. B. Clement, and a record of them appeared in the *Transactions* of the Royal Society, Series A, Vol. 201, pp. 45-61. A single brass pipe, 0.3779 in. in diameter and 6 ft. long, was used. The experiments, also, check closely the temperature factor obtained by the writer's for ordinary, or "eddy motion," flow. The range of temperatures was from 4 to 50° Cent., or 39 to 122° Fahr. The interesting point is made that the intersections of the lines for ordinary and for "stream line" flow vary as the viscosity of the water.

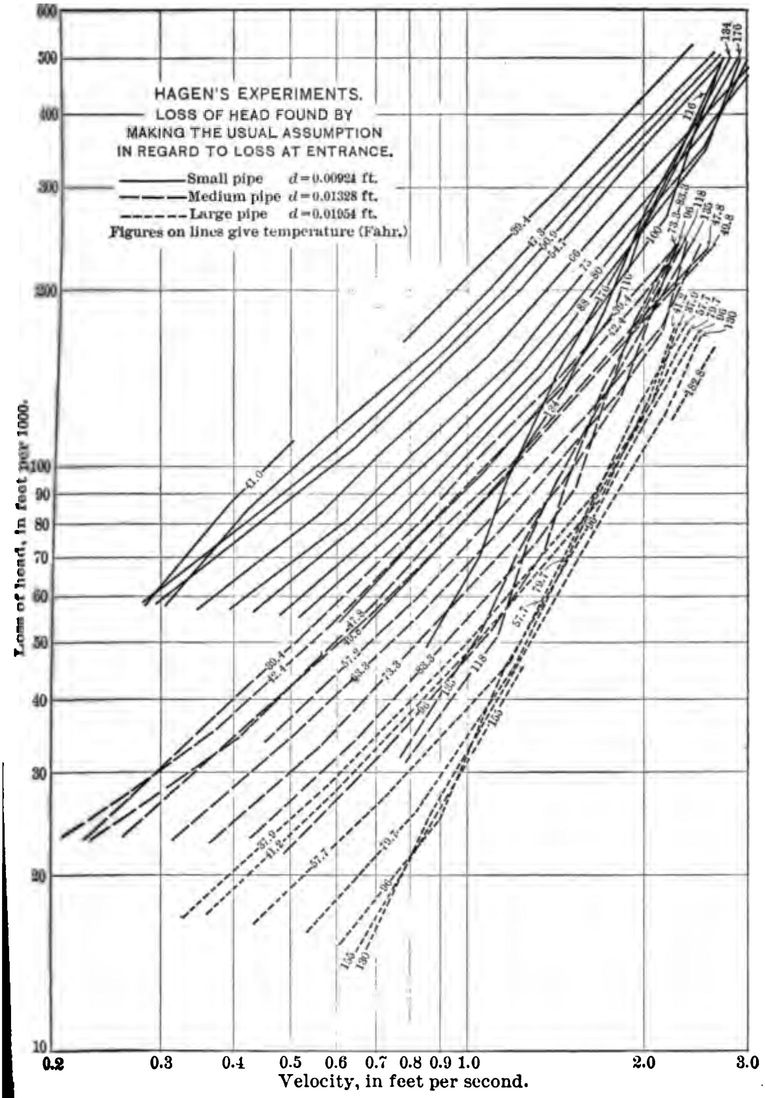
HAGEN'S EXPERIMENTS.
LOSS OF HEAD FOUND BY MAKING THE USUAL ASSUMPTION IN REGARD TO LOSS AT ENTRANCE.
Small pipe d = 0.00924 ft.
Medium pipe d = 0.01328 ft.
Large pipe d = 0.01954 ft.
Figures on lines give temperature (Fahr.)
Loss of head, in feet per 1000.
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2.0
3.0
Velocity, in feet per second.
41.0
39.4
47.8
50.5
54.7
66
73
80
89
100
110
116
134
170
182.6
57.7
79.7
90
118
135
155
130

FIG. 4.

1.75, takes place on Pipe *XVI* at a velocity of 3.5 ft. at 70
about 5 ft. for 40°. Particular attention is also called to the
of the lines, and to the fact that where the loss is proportiona
first power of the velocity the difference in head for any certain
ature interval is many times greater than where the loss is prop
to the 1.75 power of the velocity. For Pipe *XVI* a line is also c
for 75 degrees. In some pipes, Pipe *IX* for example, where t
tions show very slight differences under regular conditions of f
differences are magnified below the critical velocity.

The interesting question, however, in connection with the
velocity, is whether it is an element to be considered for the v
and pipe diameters used in practice. On the basis of the expe
on brass pipe, and assuming that there is a continuity of law wh
tended to diameters greater than those experimented upon, th
tion may be given an approximate answer. The question i
the determination of the relation between the pipe diame
the velocity at which the power of the velocity ceases to be 1.7

On Plate *XL* the points for Pipes *XVI*, *XV*, *XIII*, *IX* a
may be determined fairly well for the experiments at approxim
degrees. In addition to these points, the writers have also
dence of experiments at very low velocities on Pipe *II*. The
made early in 1901, in connection with another study in hyd
The heads were measured first with water and later with oil.
The region of critical velocity was explored thoroughly.
dence of the phenomenon was found in the first 68 ft. of pi
stream from the 5-in. pipe. For this section the ordinary flo
tion seemed to hold, down to velocities as low as 0.25 ft. per
But, in the succeeding length, corresponding to Section *A*-
present experiments, the gauges showed a decidedly unstable
of flow from 0.55 ft. per second down to about 0.35 ft. per sec
temperature of the water for these experiments was about 40 d

The critical-velocity points seem to lie approximately on a
line. Such a line was drawn, and the intersections with the
of slope, 1.75, were marked. The velocities thus determined
plotted logarithmically, with respect to the diameters, and
plotting the law for impending critical velocity at 70° de
pears to be $V = \frac{0.088}{d^{0.784}}$. This expression, being only roughl

PLATE XL.
PAPERS, AM. SOC. C. E.
MAY, 1908.
SAPH AND SCHODER ON
FLOW OF WATER IN PIPES.

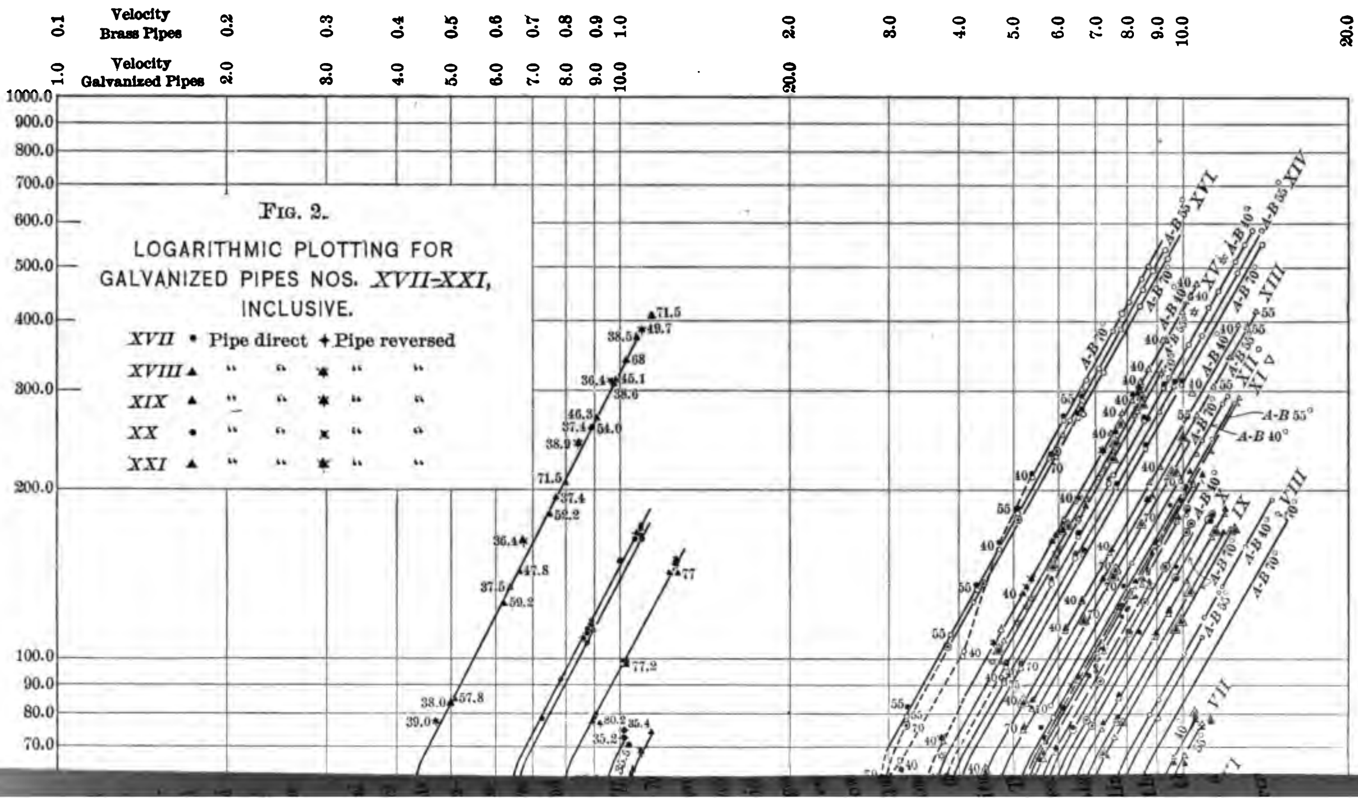

FIG. 2.

LOGARITHMIC PLOTTING FOR GALVANIZED PIPES NOS. *XVII-XXI*, INCLUSIVE.

mate, may be replaced by $V = \frac{0.1}{d^{0.75}}$ (*d*, in feet). For 40° there are only two points, namely, Pipes *XVI* and *II*, but these indicate the law expressed by $V = \frac{0.15}{d^{0.75}}$.

The older series of experiments do not include many with velocities below 1 ft. per second, and the critical velocity, therefore, is not easily determined. They seem to indicate, however, that the velocities at which ordinary conditions are established are somewhat lower than those given by the foregoing formula. However, it is seen from the formula that the change in condition takes place at very low velocities for ordinary sizes of pipe. Moreover, as appears from Pipe *II*, it seems that, for large sizes, a long length of straight pipe is necessary to allow the below-critical-velocity flow to adjust itself.

The formula given determines only the limiting point for the use of the power, 1.75. The lower points, where the loss ceases to be directly proportional to the velocity, are approximately on the same line that was drawn in the other case, but the intersections must now be taken with the lines of slope 1. The law seems to be about $V = \frac{0.045}{d^{0.85}}$. This law, however, is not of as much interest as the one applying to the upper points.

It seems that the question of critical velocity, while of considerable scientific interest, is of no practical importance to the engineer; but space has been given to its consideration on account of the fact that many scientists have been led astray in attempting to apply experimental data from very small pipes to pipes used by the engineer, simply because the change in the manner of flow was not understood.

EXPERIMENTS ON SMALL RUBBER HOSE.

It had been planned to perform a considerable variety of experiments on small hose, especially as regards the effect of curvature on the loss of head. Unfortunately, there was not enough time to follow out by a more extended experimental study the interesting and surprising results obtained.

The hose were fed from the bottom of the large drum, 1.135 ft. internal diameter and 2.167 ft. long, at the up-stream end of the 5-in. brass pipe. A bushing, a 1-in. short vertical nipple, a 1-in. reducer-elbow, a ¾-in. short nipple, a ¾-in. disc valve, and bushings into which the connecting nipples of the hose screwed, were the fittings, in their respective order. The valve was wide open during the experiments, the pressure being regulated by a 2-in. valve on the pipe leading into the drum. An air-chamber was attached to the top of the drum, and from this a connecting hose led to a two-column mercury gauge, one arm of which was open to the air. In all the experiments the hose were allowed to drop to the floor (about 6 ft. below the bottom of the drum) and laid straight (curve experiments excepted, of course) along it, except for an easy return curve of about 3 ft. diameter and a slow rise to the sharp downward curve at the discharge end. Thus the entry and discharge conditions were alike in all experiments. The level of the discharge end, with respect to the gauge scale, was determined. An extreme variation of 0.1 ft. might have occurred.

In Experiments Nos. 655 to 658 the hose was laid on a board and held in position in a horizontal plane by means of nails. The loops occurred at the ends of the board where the hose returned on itself. Except at the ends, the curves were semicircles alternating in direction of curvature, the radius being the same as when the hose was wound on the cylindrical drum.

The hose used were of three-ply, cotton-insertion, rubber tubing with smooth interior surfaces. The inside rubber lining was very heavy.

It has been the established idea that reversal of curvature introduces about twice as much loss of head as the same amount of continuous curvature, the length of pipe being the same in the two cases. The experiments on Hose *A* contradict this so flatly that it can only be concluded that there is something radically wrong with the hypothesis underlying the accepted notion. The small size of the hose might be regarded as a bar to the serious consideration of the results, but the experiments of John R. Freeman, M. Am. Soc. C. E., on 2.49-in. hose* are somewhat confirmatory. Mr. Freeman found that a full circle of 2 ft. radius (9.65 diameters) gave practically the same excess loss over straight hose as that given by four quadrants of the same radius, but introducing two reversals of curvature. In figures, the full circle gave $5\frac{1}{2}\%$ less excess loss.

In the curvature experiments on Hose *A*, the data of which are given in Table No. 8, the radius of curvature was $2\frac{15}{16}$ ins., or nearly eight diameters. The amount of curvature was purposely made large so that the effects might show up in no uncertain manner.

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* *Transactions*, Am. Soc. C. E., Vol. XXI.

TABLE No. 8.—EXPERIMENTS ON HOSE.

Hose *A*. $l = 47.93$ ft. $d = 0.370$ ins.

No. of experiment.	Temp.	Total acting *H*, feet of water.	Loss of head, in feet per 100.		Velocity.	Remarks.
647	47.5°	49.01	98.7	If laid as in 647 to 650.	8.431	$0.5\frac{V^2}{2g}$ allowed for entry; 647 to 650 hose laid with two easy curves in addition to one at beginning and one at end; 659 same; 651 to 654 hose wound in a spiral of 22 complete turns around a drum of 5¼ ins. diameter; 655 to 658, 39 curves, including 3 loops; 38 reversals of curvature: curves of same radius as in 651 to 654. In curve experiments there were curves also at beginning and end as in 647 to 650.
648	....	38.14	77.0		7.343	
649	....	27.03	54.6		6.066	
650	49	14.18	28.7		4.228	
651	....	49.10	99.9	75.5	7.248	
652	49	38.14	77.6	60.0	6.371	
653	49.5	23.87	47.5	37.3	4.886	
654	51	11.23	22.9	17.9	3.241	
655	51	49.10	99.4	84.6	7.733	
656	51.2	38.14	77.3	66.6	6.760	
657	....	25.16	51.0	45.0	5.423	
658	53.0	12.34	24.9	22.6	3.692	
659	51	49.06	98.9	....	8.418	
Hose A_1. $d = 0.380$ in.						
684	49	49.09	94.8	....	7.640	$0.5\frac{V^2}{2g}$ allowed for entry.
685	49.5	34.50	66.7	....	6.307	
686	50.5	16.73	32.4	....	4.273	$l = 50.28$ ft.
678	60	49.22	47.9	....	5.488	For A_1. Used $A + A_1$ ($l = 98.58$ ft.) and subtracted *A*, using a diagram based on Experiments 647 to 650.
679	61	36.63	35.4	....	4.674	
680	62.2	22.08	21.6	....	3.504	
681	63.5	8.86	8.7	....	2.103	
682	50	49.03	49.1	....	5.393	
683	42.4	74.05	73.8	....	6.787	
Hose B_1. $l = 11.87$ ft. $d = 0.504$ in.						
687	45	48.10	340.5	....	18.076	$0.5\frac{V^2}{2g}$ allowed for entry.
688	44.5	36.98	262.5	....	15.781	
689	44	24.79	176.3	....	12.854	
690	44	14.71	104.9	....	9.829	
691	44	7.85	56.3	....	7.078	
692	36.5	72.26	511.1	....	22.240	
Hose *B*. $d = 0.519$ in.						
693	43.5	48.80	77.1	....	7.731	For *B*. Used $B_1 + B$ ($l = 61.82$ ft.) and subtracted B_1, using a diagram based on Experiments 687 to 692.
694	43	33.68	53.2	....	6.370	
695	43.5	18.77	29.9	....	4.649	
696	44	9.71	15.7	....	3.243	
697	39	74.39	116.7	....	9.728	
Hose *C*. $l = 14.37$ ft. $d = 0.254$ in.						
698	69	49.56	325.9	....	10.750	$0.5\frac{V^2}{2g}$ allowed for entry.
699	60.5	36.63	240.8	....	9.255	
700	58.5	24.73	162.8	....	7.520	
701	58	14.48	95.6	....	5.627	
702	58	8.17	54.1	....	4.082	
703	60	4.53	30.1	....	2.973	
704	63	49.31	324.4	....	10.757	
705	50.5	49.14	322.7	....	10.815	
706	47	48.99	321.9	....	10.750	
707	44	75.00	490.7	....	13.764	
708	43	49.81	327.2	....	11.088	

The most surprising result was, that, although each series showed a most decided effect on the loss of head and the discharge, the value of n, in the formula $H = m\ V^n$, remained practically constant, the large changes being in the value of m. (There is a small increase for the case of reversed curvature.) Should this result be confirmed by further experiments along this line, it will have an important bearing on the subject of the nature of the resistances which cause loss of head in a pipe. For it would appear that the value of the exponent, n, is determined solely by the nature of the inner surface of the pipe, while m depends on the distribution of the velocities throughout the cross-section, and on the temperature.

The fact that the reversed curves caused only about half as much increase of resistance as the continuous curves is certainly something to think about. It is to be hoped that further and more elaborate experimentation may bring out the laws which govern the resistances in various cases of flow in curves. It may not be out of place to suggest the bearing of such studies, not only on sinuous supply mains, but also on the flow in turbine wheels.

The results of the experiments on the hose when laid with easy curves and tangents are of interest in their relation to the results of the brass and galvanized pipes. The values of m and n, as obtained from a logarithmic plotting, are as follows:

Hose.	Diameter.	m, in feet per 100 ft.	n.
A	0.370 in.	2.24	1.78
B	0.519 "	1.82	1.82
B_1	0.504 "	1.32	1.91
C	0.254 "	4.2	1.81

The experiments on B_1 were made with high velocities (7 to 22 ft. per second), and it is probable that the usual allowance for entry loss is insufficient, the velocities in the ¾-in. fittings being considerable. The high value of n and the low value of m bear out this statement. Hose B is free from this objection.

Especial attention is called to the low values of n for Hose A, B and C.

EXPERIMENTS ON ENTRY LOSS.

The writers made some experiments to investigate the commonly accepted value for loss of head due to a square-edged entry. The experiments were made on Pipes *II* and *X*. For Pipe *II* the cycloidal mouthpiece at the junction of the 5-in. brass pipe was replaced by a specially turned brass bushing with a plain inside face. The opening in this is 2.096 ins. in diameter, the same as Piezometers *A* and *B*. Section *A*-*B* of Pipe *II* was moved up stream next to the 5-in. pipe, and Piezometer *A* was inserted between the first two pipe lengths of the section. The slight ridge where the pipe was screwed up against a shoulder in the bushing was smoothed down. The entry length of Pipe *X*, likewise, was screwed into a brass bushing, giving a square-edged entry. An air-chamber was substituted for the end air-cock on top of the header, and the pressure was taken from this. The diameter of the header (2½ ins.) is so large, relatively to Pipe *X*, that practically static conditions prevailed. For Pipe *II*, however, the effect of the small velocity in the 5-in. pipe on the up-stream piezometer has been considered.

The results are given in Table No. 9. It will be seen that the values for Pipe *II* check fairly the accepted value, $0.5 \frac{V^2}{2g}$. Pipe *X*, however, gives lower indications, the values seeming to average about $\frac{1}{3} \frac{V^2}{2g}$. These results indicate the possibility of a higher value than $0.5 \frac{V^2}{2g}$ for pipes of commercial size.

EXTENT OF ENTRY DISTURBANCES IN SUCCEEDING TANGENT.

The experiments on Pipe *II* seem to show that the piezometric effects of a square-edged contraction do not extend 78 diameters down stream. Section *A*-*B* gives normal results. In Pipe *X*, on the other hand, Experiments 666 to 670 show that 43 diameters are not sufficient, for both the high results for entry loss and the low results for the loss in Section *A*-*B* show that Piezometer *A* was in the abnormal-flow region. With these results the evidence obtained by the writers on the effects of curves* should be compared.

PRACTICAL DEDUCTIONS.

The engineer will ask: "Of what use are refined laboratory experiments on pipes with conditions never met in practice?" Waiving the answer, for the present, let us examine the evidence to be had on the general subject of the flow of water in pipes.

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* *Transactions*, Am. Soc. C. E., Vol. XLVII, p. 317.

A.—Pipe *II.*

No. of experiment.	Temperature.	Observed head in water in section between Piezometer in 5-in. pipe to piezometer *A* in 2-in. pipe. Double centimeters.	Observed head in Section *A-B* in water. Double centimeters.	Loss of head due to entry, *A-B* considered normal. Double centimeters.	Velocity in Pipe *II*, in feet per second.	Remarks.
733...........	36.2°	22.89	39.92	$3.50 = 0.475 \frac{V^2}{2g}$	5.611	The up-stream section consists of 3.5 ft. of 5-in. pipe and 13.543 ft. of 2-in. pipe. Length *A-B* = 44.635 ft. Square-edged contraction. *A-B* is normal, since loss is the same as in other experiments.
734...........	36.0	16.85	29.95	2.48 = 0.454 "	4.811	
735...........	37.0	5.46	10.34	0.79 = 0.508 "	2.594	
736...........	36.8	10.61	19.40	1.50 = 0.453 "	3.761	

B.—Pipe *X.*

No. of experiment.	Temperature.	Observed head in Section Header-*A*. Double centimeters of mercury.	Observed loss of head in Section *A-B*. Double centimeters of mercury.	Loss due to entry alone, *A-B* considered normal in each case. Double centimeters of mercury.	$\frac{V^2}{2g}$ in double centimeters of mercury.	Velocity, in feet per second.	Loss in *A-B*, in feet per 100. Water.	Remarks.
660...........	54.5°	14.16	13.93	$0.71 = 0.333 \frac{V^2}{2g}$	2.13	10.628	170.7	In 660 to 665: l (entry) = 5.459 ft. l (*A-B*) = 6.719 "
661...........	56.1	21.20	20.56	1.16 = 0.348 "	3.84	13.319	252.0	
662...........	57.8	8.50	8.48	0.40 = 0.330 "	1.21	8.011	103.9	
663...........	47.8	8.60	8.64	0.40 = 0.339 "	1.18	7.924	105.9	
664...........	48.8	14.18	14.04	0.69 = 0.332 "	2.08	10.506	172.0	
665...........	50.9	5.38	5.47	0.23 = 0.324 "	0.71	6.186	67.0	
666...........	63.9	5.27	13.62	0.81 = 0.378 "	2.14	10.672	167.1	In 666 to 670: l (entry) = 1.144 ft. l (*A-B*) = 6.719 "
667...........	46.2	5.17	13.94	0.76 = 0.372 "	2.04	10.407	170.8	
668...........	48.0	3.63	9.93	0.54 = 0.386 "	1.40	8.611	121.7	
669...........	50.4	2.11	6.01	0.31 = 0.398 "	0.78	6.456	73.6	
670...........	52.0	7.94	20.38	1.22 = 0.376 "	3.25	13.150	249.7	
671...........	79.5	15.06	10.05	0.67 = 0.330 "	2.03	10.896	151.9	In 671 to 676: l (entry) = 6.720 ft. l (*A-B*) = 5.462 "
672...........	82.9	22.63	14.89	1.09 = 0.339 "	3.22	13.076	225.2	
673...........	52.8	15.44	10.45	0.68 = 0.358 "	1.90	10.047	157.6	
674...........	51.2	4.55	3.18	0.16 = 0.334 "	0.48	5.070	47.9	
675...........	51.8	9.26	6.34	0.38 = 0.352 "	1.08	7.588	95.6	
676...........	53.0	24.54	16.36	1.21 = 0.378 "	3.20	13.044	246.6	

Wherever possible, the original records of experimental data have been consulted. By the method of logarithmic plotting of losses of head with velocities, some 150 series of experiments have been investigated, and the values of *m* and *n* determined. The writers early came upon the necessity of either considering all the evidence, or else of running the risk of making errors in judgment and perhaps of allowing partiality more play than desirable in such a study.

In the preliminary plotting, all experiments which had been investigated were considered. In the final plotting many are left out. But the writers can state conscientiously that the conclusions are not altered thereby. The experiments omitted from the plotting are mostly old German and English experiments, the exact conditions and methods of measurement being generally unknown. If they were included (and the writers were sorely tempted to do so on account of the indications of most of them) they could not be given much weight, no matter what the indications, on account of this uncertainty.

Considering now the general subject, there is presented on Plate XLI a logarithmic plotting of the values of *m* (as determined from plottings of each series of experiments), with the diameters of the pipes. It will be noticed that the values of *n* are marked near the points. These serve as descriptions of the nature of the pipe, from the experimental standpoint. In addition to this, the material is also indicated. This latter is the engineer's description, and is usually far from complete, but, with the assistance of the knowledge of the value of *n*, the indications can be discussed more rationally.

It will be seen that the range of pipe diameters varies from $\frac{1}{10}$ in. to 12 ft. A very casual examination will show that all the points lie in a comparatively narrow zone and hence indicate some exponential relation.

First of all, the line best fitting the points for the writers' brass pipes has been drawn. On account of the very considerable effect of temperature on the value of *m*, the middle points, for 55°, have been used. There are plotted, also, the points for 40° and 70°, but these are not used, except to convey to the eye the magnitude of temperature effect.

The variations in the value of *n* for the seamless-drawn brass pipes (all apparently of the same degree of roughness), have been noted. Hence, corresponding indications due to the values of *m* are to be expected, and the variations from the line which has been drawn seen on Plate XLI.

One feature will be noticed at once. The line for the brass pi forms the lower limit of the zone in which all the plotted points This is another way of stating that these pipes represent the extre of smoothness and ideal conditions. The equation of this line, n lecting the third place of decimals in the exponent, is

$$m = \frac{0.296}{d^{1.25}}$$

in which m is in feet per 1 000 ft., and d is in feet. Placing this va in the general expression, $H = m\ V^n$, and using the average valu n, there results

$$H = \frac{0.296}{d^{1.25}}\ V^{1.75}$$

in which H is in feet per 1 000 ft., and d is in ft. This expressi then, gives the loss of head for extremely smooth pipes laid straig But this is not all. There occur maximum variations in the valu m of approximately 7% each way from the line of this equati Hence, it is seen that this variation must be taken into acco There is also an effect due to the variation in n, and this may bec considerable for high velocities. It is not as prominent a facto the smooth brass pipes as in pipes used in practice, but it is poin out as something to be borne in mind. The writers might wish many hydraulic engineers have wished, that there was less uncerta in designing pipe systems, but, since it exists, its probable ex should be made an integral part of any formula. Therefore, concern smooth brass pipes, under nearly ideal conditions, it becomes neces to write, for the general expression,

$$H = \frac{0.296}{d^{1.25}}\ V^{1.75} \pm 7\%.$$

Looking at this, and remembering the long months of monoto experimentation, the care taken to have everything accurate, temperature factor, which alone may cause a variation as large as t given above, the writers feel that it nevertheless represents a step advance; for while it does not tend to make hydraulic science more exact, it at least helps to make it safer in application.

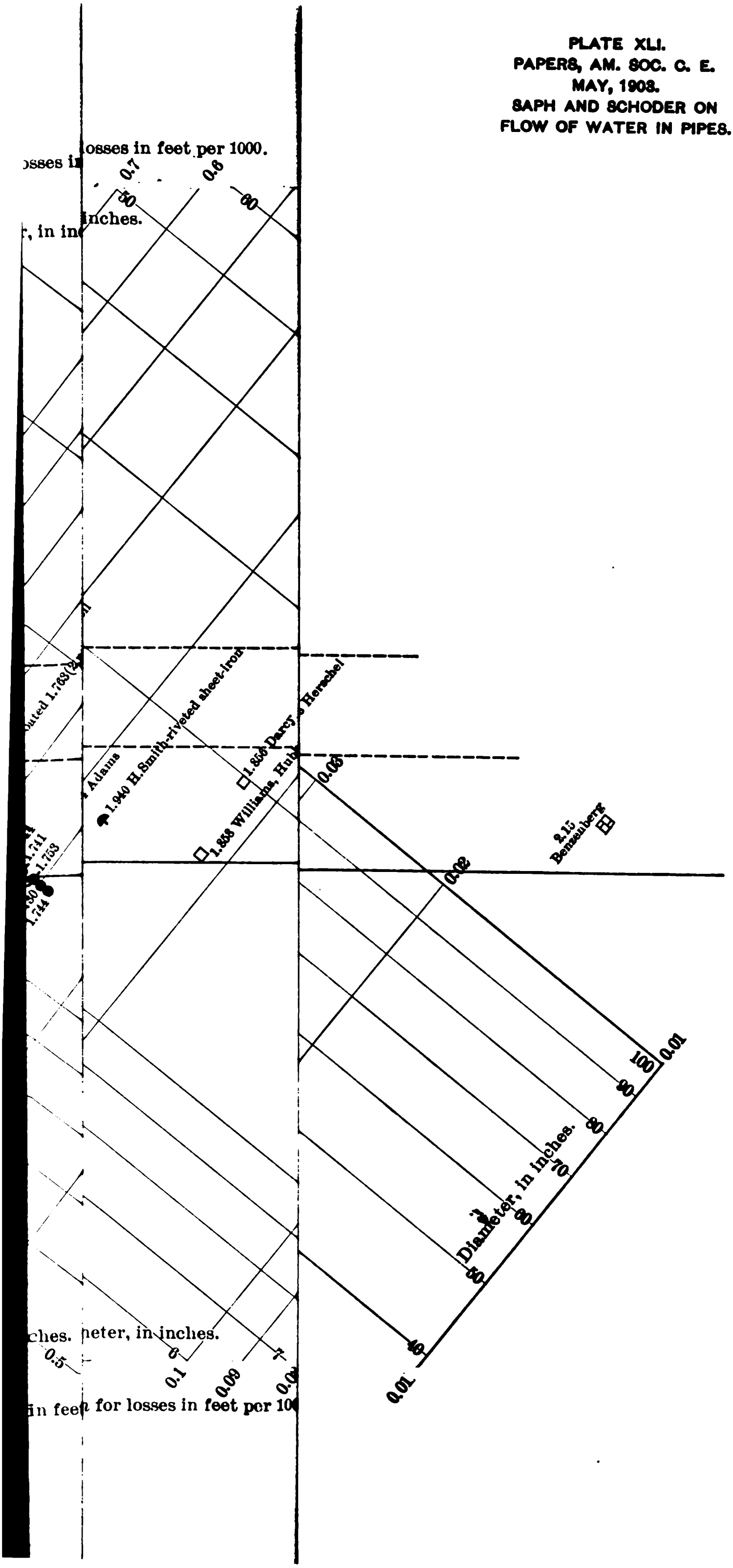
Modifying the above formula, there is obtained the expression

$$V = 104\ s^{0.57}\ d^{0.71} \pm 4\%;$$

or, using the "hydraulic mean radius,"

$$V = 279\ s^{0.57}\ R^{0.71} \pm 4\%.$$

PLATE XLI.
PAPERS, AM. SOC. C. E.
MAY, 1903.
SAPH AND SCHODER ON
FLOW OF WATER IN PIPES.



In these last expressions s is the hydraulic slope, in feet per foot of length. In these expressions temperature effect has not been included. As has been stated, the effect is to vary the loss of head about 4% for a change of 10° Fahr. Owing, however, to the uncertainty regarding the effect on the rougher pipes used in practice, the writers feel that its insertion might be misleading.

Commercial Pipes.—Considering now the kinds of pipes used in practice, the writers desire, first of all, to call attention again to the indications of their experiments on galvanized-iron pipes. It is seen that, not only is there a variation in the exponent, n, from 1.80 to 1.96, but that Pipe *XVIII*, although having but two-thirds the internal sectional area of Pipe *XVII*, nevertheless, has a decidedly greater discharging capacity per unit of sectional area, or, in other words, offers less resistance to the flow, for the same velocity. Nevertheless, to the eye, all the pipes appear to be of the same degree of roughness. These two facts throw a strong illumination on the hitherto unaccountable variations in the results of standard experiments.

Although Darcy's experiments showed clearly the effect of variations in roughness which were easily perceptible, and which could be described so as to be recognized by ordinary inspection, yet the hydraulic literature of the past decade or two bears out the statement that the extent of the effects of variations in character of surface which practically defy detection have not been appreciated. Otherwise, how can we account for statements by well-known engineers that certain formulas will give results within 2 or 3% of the truth?

Turning again to the general plotting on Plate XLI, we have to consider the experiments which lie in the zone, one of the limits of which is the line already discussed. Attention should be called to the fact that some of the series of experiments do not show much harmony in the indications, and that, therefore, it is a matter of judgment just where to draw the line on the logarithmic plotting of the losses of head and velocities. Hence, the values of m and n have the effects of the personal equation. But the two values, taken together, represent, as well as may be, the conditions in the range of the series under investigation.

It will be seen that the variations in the value of the exponent, n, are greater than the variations found by the writers for the galvanized pipes. Outside of the heavily tuberculated and riveted pipes, this variation is from 1.74 to 2.00. The exponents given for the series by Herschel and by Darcy represent fairly the point which has been made. The values of m and n for the 30-in. pipe of Williams, Hubbell and Fenkell have been furnished to the writers by Professor Williams. They are the result of a plotting of the original data, on their own merits.*

It will be seen that a line parallel to the one already drawn will represent fairly the other limit of the zone. The experiments which determine this line are those by Darcy on tuberculated, cast-iron pipe, and some of those by Herschel and some by Marx, Wing and Hoskins on riveted-steel pipe. Noble's 44-in. wood-stave pipe also comes close to this line. The equation is $m = \frac{0.687}{d^{1.25}}$. As for the general equation, we have a variation in the value of n from 1.67 to 1.99. The value, 1.67, from Herschel's 36-in. pipe, is lower than anything else on the plate. The writers consider that this value is questionable, although it is what the series of experiments gave.† Hence, the variation in n will be considered as from 1.82 to 1.99, and the general formula becomes

$$H = \frac{0.687}{d^{1.25}} V^{1.82 \text{ to } 1.99}$$

in which H is in feet per 1 000 ft., and d is in feet. Converting this into the other form, there appears

$$V = 54.7 \text{ to } 38.9\, s^{0.55 \text{ to } 0.50}\, d^{0.69 \text{ to } 0.63}.$$

Using the hydraulic radius, this becomes

$$V = 142 \text{ to } 93\, s^{0.55 \text{ to } 0.50}\, r^{0.69 \text{ to } 0.63}.$$

The two sets of equations thus far given, then, may be regarded as giving the two extremes, the first for very smooth pipes laid very straight, and the second for a considerable degree of tuberculation (or an inner surface comparable to it).

But neither one of these formulas represents the great majority of cases which confront the hydraulic engineer. These latter are included in the zone bounded by the full line and the middle line on Plate XLI. The values of n vary from 1.74 to 2.00. The equation is $m = \frac{0.296 \text{ to } 0.469}{d^{1.25}}$ and the general expression becomes

$$H = \frac{0.296 \text{ to } 0.469}{d^{1.25}} V^{1.74 \text{ to } 2.00}.$$

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* Not as described in *Transactions*, Am. Soc. C. E., Vol. XLVII, pp. 113 and 188.

† Mr. Herschel, himself, questions the reliability of seven out of eight experiments.

In the second form, this gives

$$V = 107 \text{ to } 46\, s^{0.57 \text{ to } 0.50}\, d^{0.72 \text{ to } 0.62},$$

or

$$V = 289 \text{ to } 109\, s^{0.57 \text{ to } 0.50}\, r^{0.72 \text{ to } 0.62}.$$

This last series of equations may be said to represent the range of results which the best modern experimenters have obtained on actual pipe lines in service or ready for service. The range of values is large, but the writers do not desire to narrow the analysis any more at present. Their purpose will have been accomplished if this paper serves to bring out some of the factors which have been neglected or not fully appreciated in the past. Above all, it is hoped that a vigorous discussion, in the true scientific spirit, may result, and that less of personality and more of the truth may appear than is wont to be the case.

The experiments, the data of which are given in this paper, were performed between February 24th, 1902, and February 25th, 1903. Experiments Nos. 25 to 771 were performed after July 10th, 1902. Earlier preliminary experiments, during 1901, are noted in the paper.

The original notes and the data tabulated from them are on file in the Hydraulic Laboratory of the College of Civil Engineering of Cornell University, where they are available for examination at any time.

APPENDIX.

Discussion of Theories.

The writers do not venture, at present, to give any hypothesis showing why the exponent for smooth pipes should be 1.75. They merely give it as an experimental fact.

There has been an attempt, however, to explain the matter by saying that the resistance is really composed of two parts, one due to the true friction on the pipe wall, and depending on the first power of the velocity, and the other due to the impacts of the particles on each other and depending on the square of the velocity. The theory states that the composition of these two parts will be equivalent to one term with the power, 1.75. The formula which expresses this idea is $H_f = x V + y V^2$, x and y being coefficients, the values of which are to be determined. Since this formula (or its many modifications) has been used extensively, and since it expresses the idea put forth above, the writers have investigated it, to see, if in this form or in any of its modified forms, it can be made to represent accurately a series of experimental results.

In order to determine the values of the coefficients x and y, at least two methods have been in vogue. The first—the method of least squares—is allowable only on condition that the coefficients are proved to be constants, otherwise the formula derived and its limits of error apply only to those experiments which are used, and its application may be limited. The second method—a graphical one—is for this reason more general, but in most cases in which it has been applied it has not been carried to completion. It consists in obtaining first a series of equations in x and y by substituting the observed values of H_f and V. These equations may be represented by straight lines, and, if x and y are constants, all these lines should pass through one point the coördinates of which are the desired values. Ordinarily, however, it is said that the lines will not pass through a point, but will enclose a small area, and in such a case the coördinates of the center of gravity of the area are the values of x and y. This method has been applied by the writers to a number of series of experiments, and in no case, if a suitable scale was used, did the lines pass through a point or enclose an area. In every case the lines formed a series of tangents to a regular curve similar to one branch of a hyperbola, and to which the axes were apparently asymptotic. This showed clearly that x and y, instead of being constants, were really functions of V. Further investigation also showed that x and y were always such functions of V that the final result was that obtained by a direct logarithmic plotting.

For example, if the logarithmic plotting leads to $H_f = m\ V^{1.75}$, this method leads to $H_f = x\ V + y\ V^2 = (a\ V^{0.75})\ V + (b\ V^{-0.25})\ V^2 = (a + b)\ V^{1.75}$. If it is admitted that $H_f = m\ V^{1.75}$ represents accurately the relation, it can also be proved mathematically that any attempt to express the relation in two terms will lead to variable coefficients and finally result in the same way as shown above.

A more general expression, $H_f = a\ V + b\ V^2 + c$, is sometimes used. It can be shown, also, that a, b and c will be functions of V. In a number of cases where this form has been used, negative values of c have resulted when a, b and c have been found by least squares. These, surely, are not easily interpreted.

The foregoing investigation shows that the explanation given for the presence of the exponent, 1.75, and which is expressed by the Prony formula, $H_f = x\ V + y\ V^2$, by those who hold to this idea, is questionable. It also shows that all formulas which involve V^2 in any way will also involve in general a coefficient depending on V. The exception consists in the fact that a very few experiments on pipes do show an exponent of 2. As is readily seen, we thus account for the variables f and c in the familiar formulas, $H = 4 f \frac{l}{d} \frac{V^2}{2 g}$ and $V = C \sqrt{r\ s}$. Of all the forms yet advanced, the only one which will represent the conditions at all accurately is the one the writers have adopted, $H_f = m\ V^n$, and n is to be determined for each series of experiments. It will be seen, of course, that the formula, $V = C\ r^x\ s^y$, is merely a modification of $H_f = m\ V^n$, m being dependent on d.

The writers, for a time, had the idea that the reason for the exponent of V being different from 2 might lie in not distinguishing between the square of the mean velocity and the sum of the squares of the individual velocities throughout the cross-section of the pipe. (They fully recognized, however, the fact that the idea of flow in parallel filaments is merely a theoretical convenience.) They therefore have investigated the question as to what power of V_m will give the same result as $\frac{\Sigma\ V^2}{\pi\ r^2}$. The supposition that the velocity curve is an ellipse, led to results which indicated that their view was correct. The exponent of V_m had a value of 2 for a velocity of 8 ft. per second and decreased for lower velocities. For a velocity of 1 ft. per second, many peculiarities in the values of the exponent were shown similar to those which the present experiments show in the critical-velocity transition stage. However, the best test of theory is experiment, and when a series of Pitot tube traverses was investigated, there was found the experimental fact that the relation, $V_m^2 = \frac{\Sigma\ V^2}{\pi\ r^2}$, holds almost exactly in every case. While proving that the proposed explanation was